

# Phase-aware Statistics and their Application to Storm Surge Forecasting

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# Outline

- Background material
- Phase-aware theory
- Storm surge forecasting applications

Background

# Hurricane Sandy



# Ensemble Forecasting

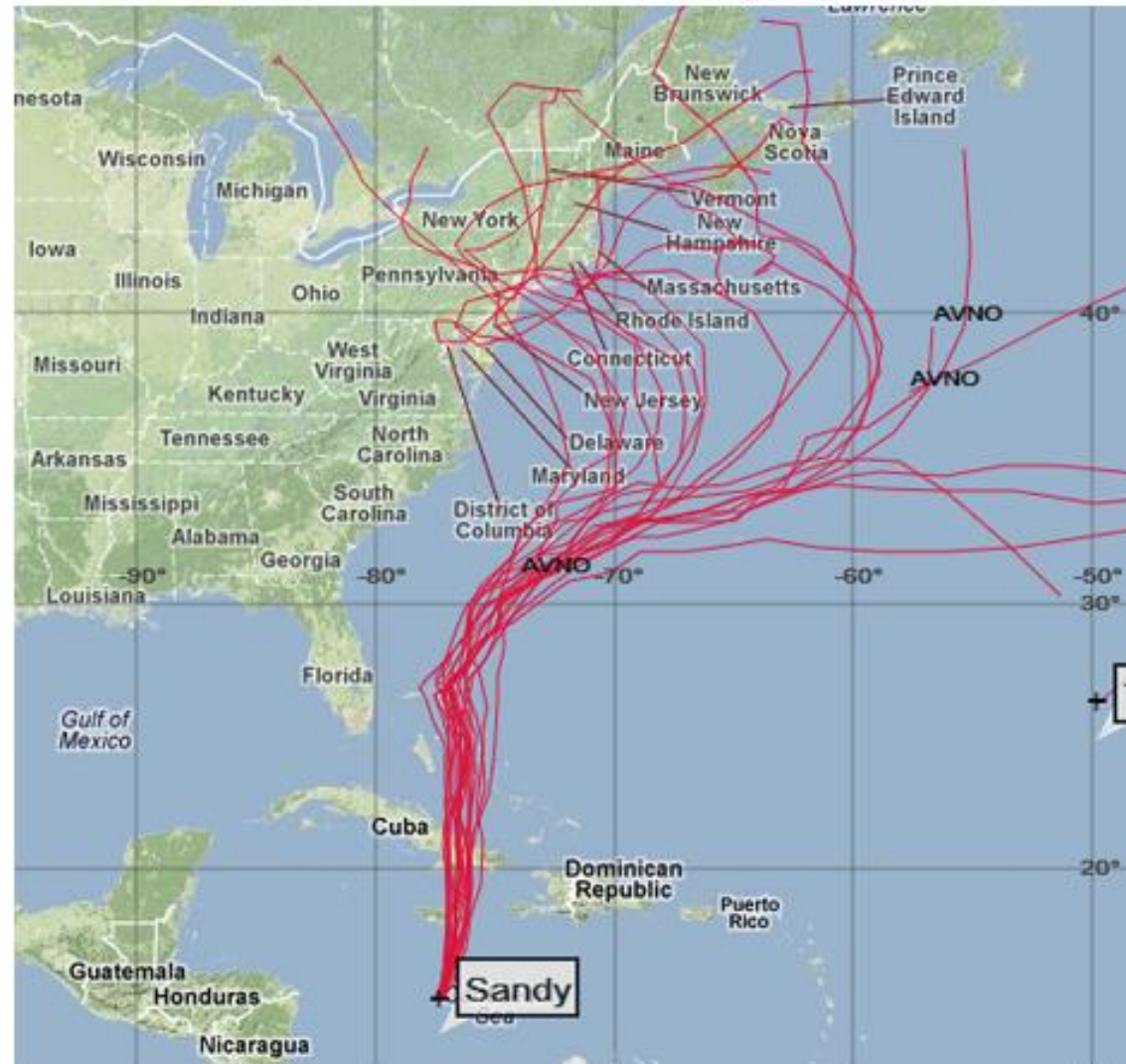
- Estimates the uncertainty of a weather forecast using multiple predictions.
- Each prediction is called an ensemble member.
- The collection of ensemble members is the sample space or ensemble system.
- The method contrasts with deterministic forecasts.
- Deterministic forecast: The forecast high is 75°F.
- Ensemble forecast: The forecast high is likely to be between 70°F and 80°F.

# Rolling a Die

- Sample Space = {1,2,3 4,5,6}
- The members of the sample space are the ensemble members.
- The ensemble members represent what could happen when the die is rolled.

# Real-world Example: Hurricane Sandy

GEFS Forecast Tracks for Hurricane Sandy 00 UTC 24 Oct 2012



# The Ensemble Mean

- Commonly, an ensemble mean is reported.
- The ensemble mean is the mean of all ensemble members.
- The ensemble mean suppresses the unpredictable aspects associated with the individual members.
- Forecast error is often measured relative to the ensemble mean.



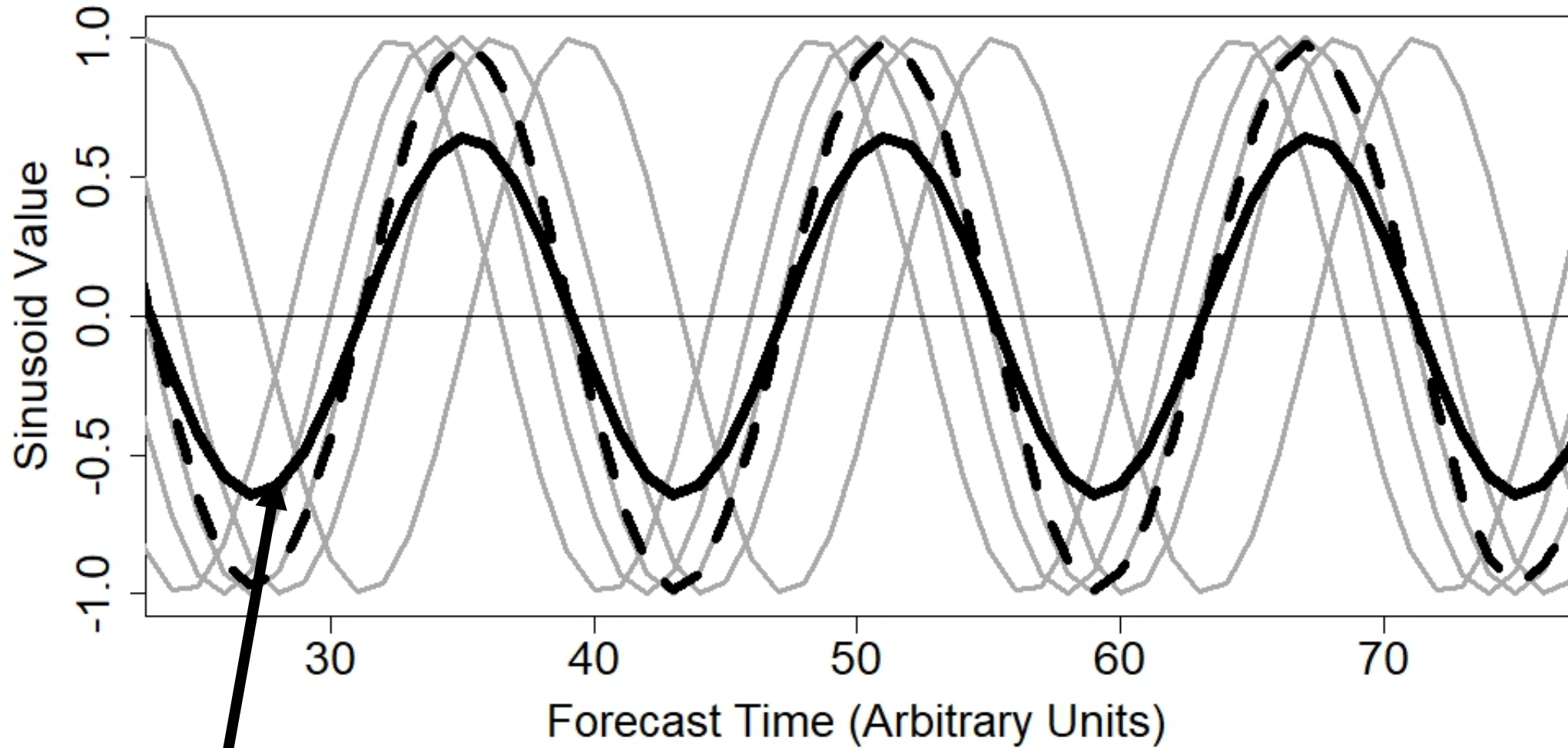
# One Way of Measuring Forecast Error

- Root mean square error =  $\sqrt{(ensemble\ mean - Observation)^2}$
- Factors contributing to forecast error:
  - Forecast uncertainty
  - Imperfect model physics
  - Initial conditions
- Research question: Is the ensemble mean the best quantity on which to base forecast error?

# The sinusoid Conundrum: Experimental Set up

- Ensemble System =  $\{\sin(\omega t + \theta_1), \sin(\omega t + \theta_2), \dots, \sin(\omega t + \theta_5)\}$
- **Feature 1**- Each sinusoid has an amplitude equal to 1 (no intensity uncertainty).
- **Feature 2** - Phases are drawn from a normal distribution with mean 0 and standard deviation  $\pi/3$  (large timing uncertainty).
- Each ensemble member is a possible outcome for the “observation.”
- The observation is an additional randomly generated sinusoid.

# Sinusoidal Ensemble System



Ensemble Mean Amplitude < 1!

# Key Findings

- The ensemble mean can lead to intensity error even if there is no intensity uncertainty!
- Timing differences among ensemble renders the ensemble mean unrepresentative of the ensemble system.
- Unrepresentativeness means that the ensemble mean has characteristics differing from the individual ensemble members.
- The ensemble mean flattens out as timing uncertainty increases.
- How can we remedy these drawbacks?

# Phase-aware Theory

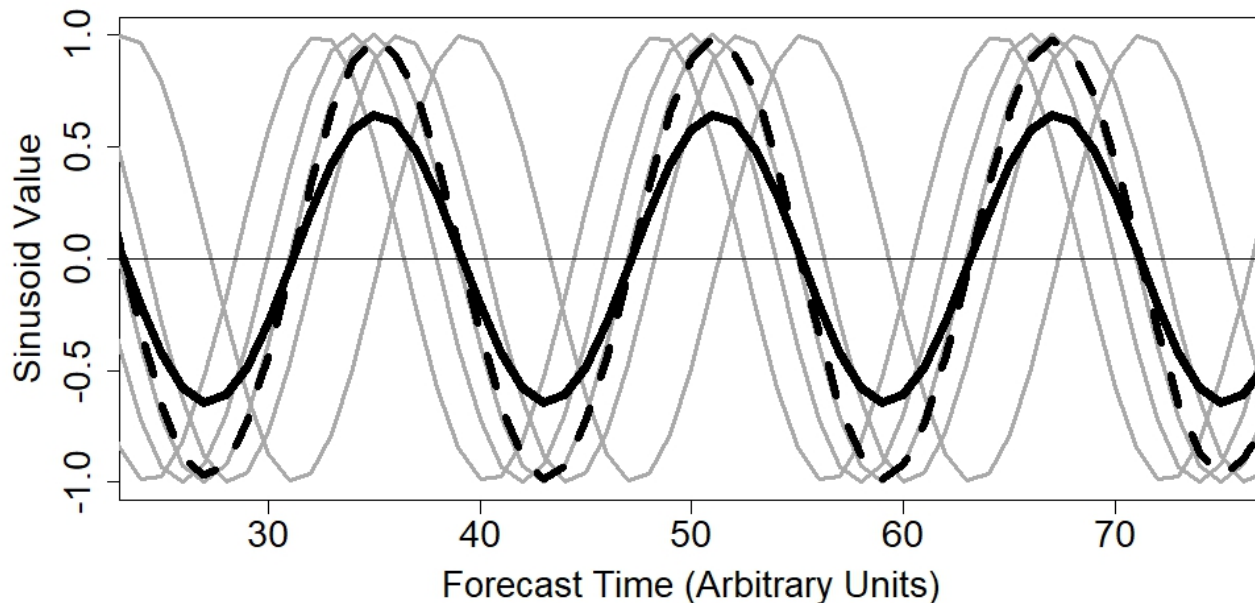
# Motivation

$$\text{Ensemble System} = \{\sin(\omega t + \theta_1), \sin(\omega t + \theta_2), \dots, \sin(\omega t + \theta_5)\}$$

$$\hat{A} = \frac{A_1 + A_2 + \dots + A_5}{5}$$

$$\hat{\theta} = (\text{circular}) \text{ mean of phases}$$

$$\hat{X} = \hat{A} \sin(\omega t + \hat{\theta})$$



Research question: Can we do this procedure for arbitrary ensemble systems?

# The Wavelet Transform

**Wavelet Transform of Time Series**

**Modulus – indicates how strongly a time series fluctuates**

**Phase – describes how and when the time series fluctuates. Periodic? Rises and Falls?**

**Wavelet Coefficient = modulus \* phase**

Inverse Wavelet Transform

**Original Time Series**

# Phase-aware Mean: The Recipe

**Step 1. Compute Wavelet Transform of each Ensemble Member**



**Step 2. Compute Arithmetic Mean of Modulus (Intensity)**



**Step 3. Compute Circular Mean of Phase (Timing)**

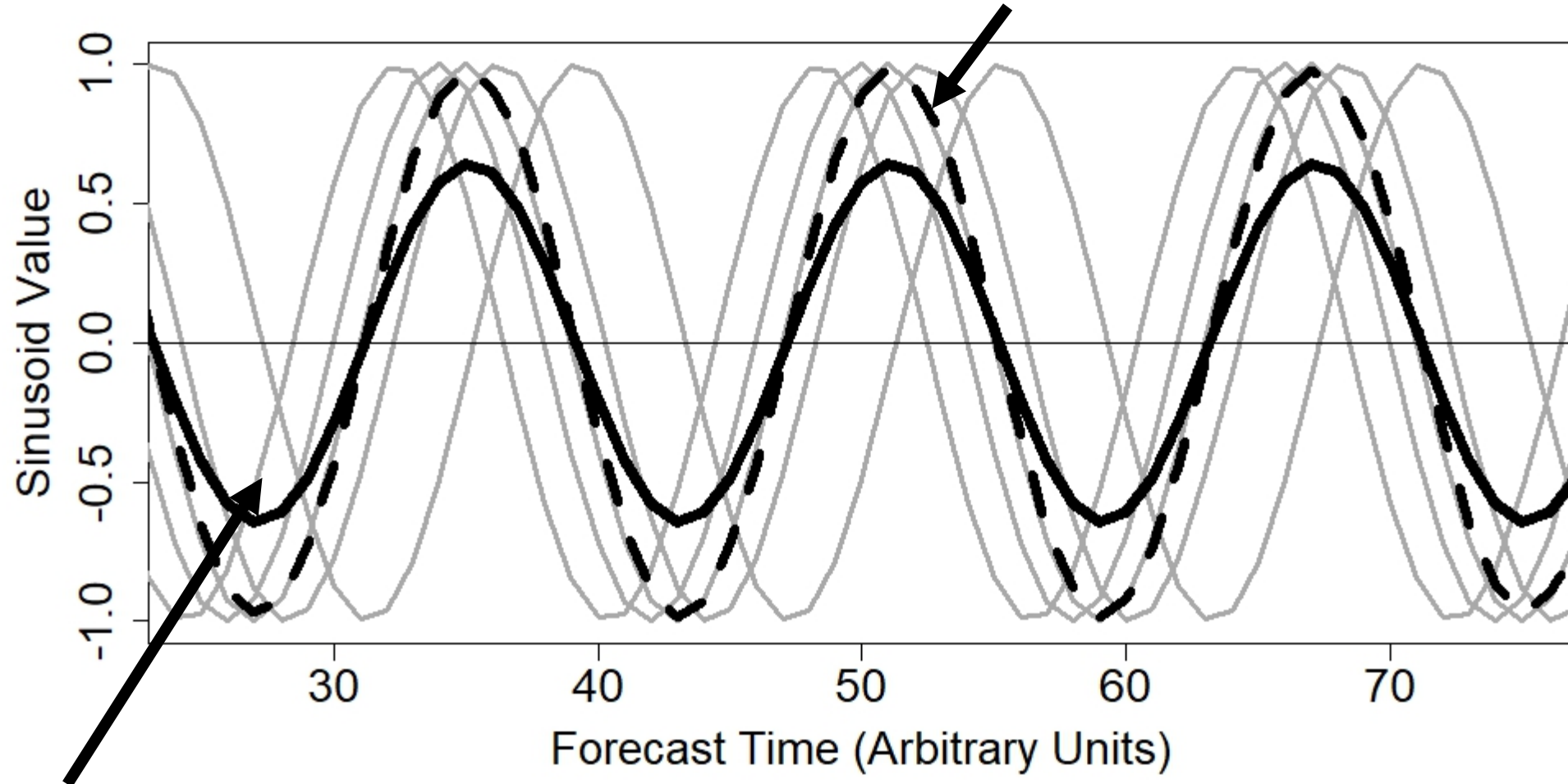


**Step 4. Compute Inverse Wavelet Transform of  
mean wavelet coefficient = (mean modulus)\*(circular mean phase)**



# Phase-aware mean Example

Sinusoid with amplitude = 1 and with phase equal to mean of all phases



Ensemble Mean Amplitude < 1!

# Phase Aware Extensions

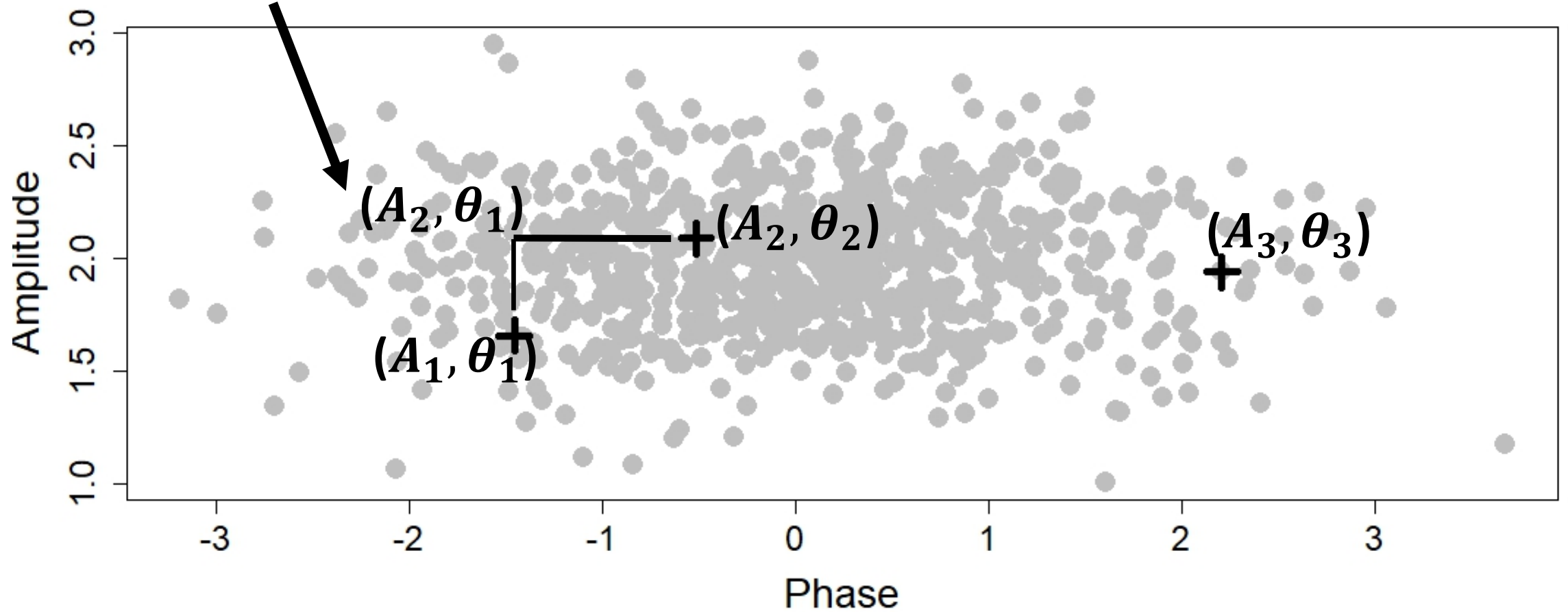
- An ensemble member can perfectly predict timing but poorly predict intensity.
- Conversely, another ensemble member can perfectly predict timing but poorly predict intensity.
- Can we create an ensemble member that perfectly predicts timing and intensity?

# Phase-Aware Extensions

- Suppose our ensemble system comprises 3 sinusoids with amplitudes  $A_1, A_2, \dots, A_3$  and phases  $\theta_1, \theta_2, \dots, \theta_3$  drawn from normal distributions.
- This ensemble system assumes that one ensemble member will predict both timing (phase) and intensity (amplitude) correctly.
- Is this a good assumption?

# Phase-Aware Extensions

It is possible!



# Phase-Aware Extensions

$$A_1 \sin(\omega t + \theta_1)$$

$$A_1 \sin(\omega t + \theta_2)$$

$$A_1 \sin(\omega t + \theta_3)$$

$$A_2 \sin(\omega t + \theta_1)$$

$$A_2 \sin(\omega t + \theta_2)$$

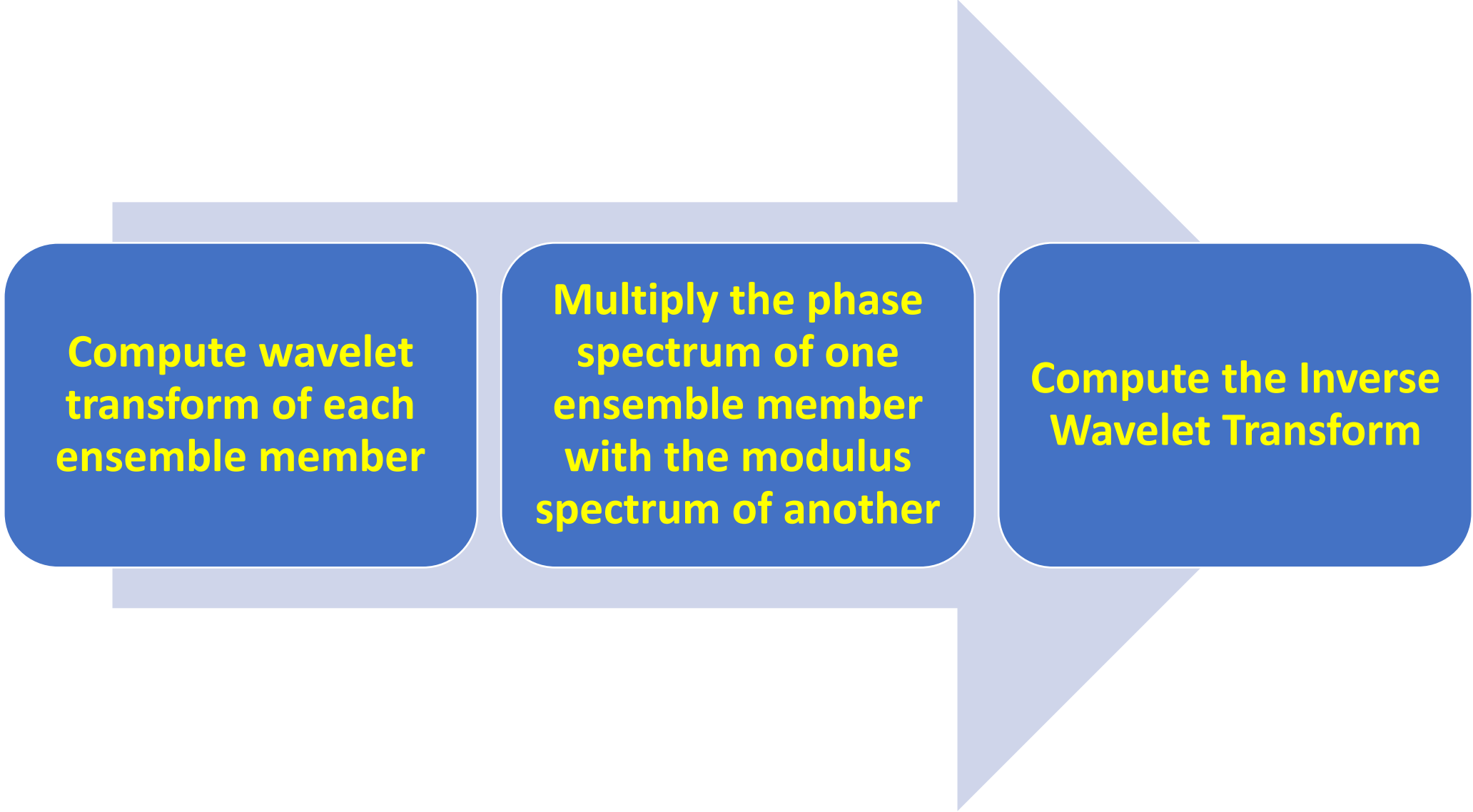
$$A_2 \sin(\omega t + \theta_3)$$

$$A_3 \sin(\omega t + \theta_1)$$

$$A_3 \sin(\omega t + \theta_2)$$

$$A_3 \sin(\omega t + \theta_3)$$

# Phase-aware extension Method



```
graph LR; A[Compute wavelet transform of each ensemble member] --> B[Multiply the phase spectrum of one ensemble member with the modulus spectrum of another]; B --> C[Compute the Inverse Wavelet Transform];
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**Compute wavelet transform of each ensemble member**

**Multiply the phase spectrum of one ensemble member with the modulus spectrum of another**

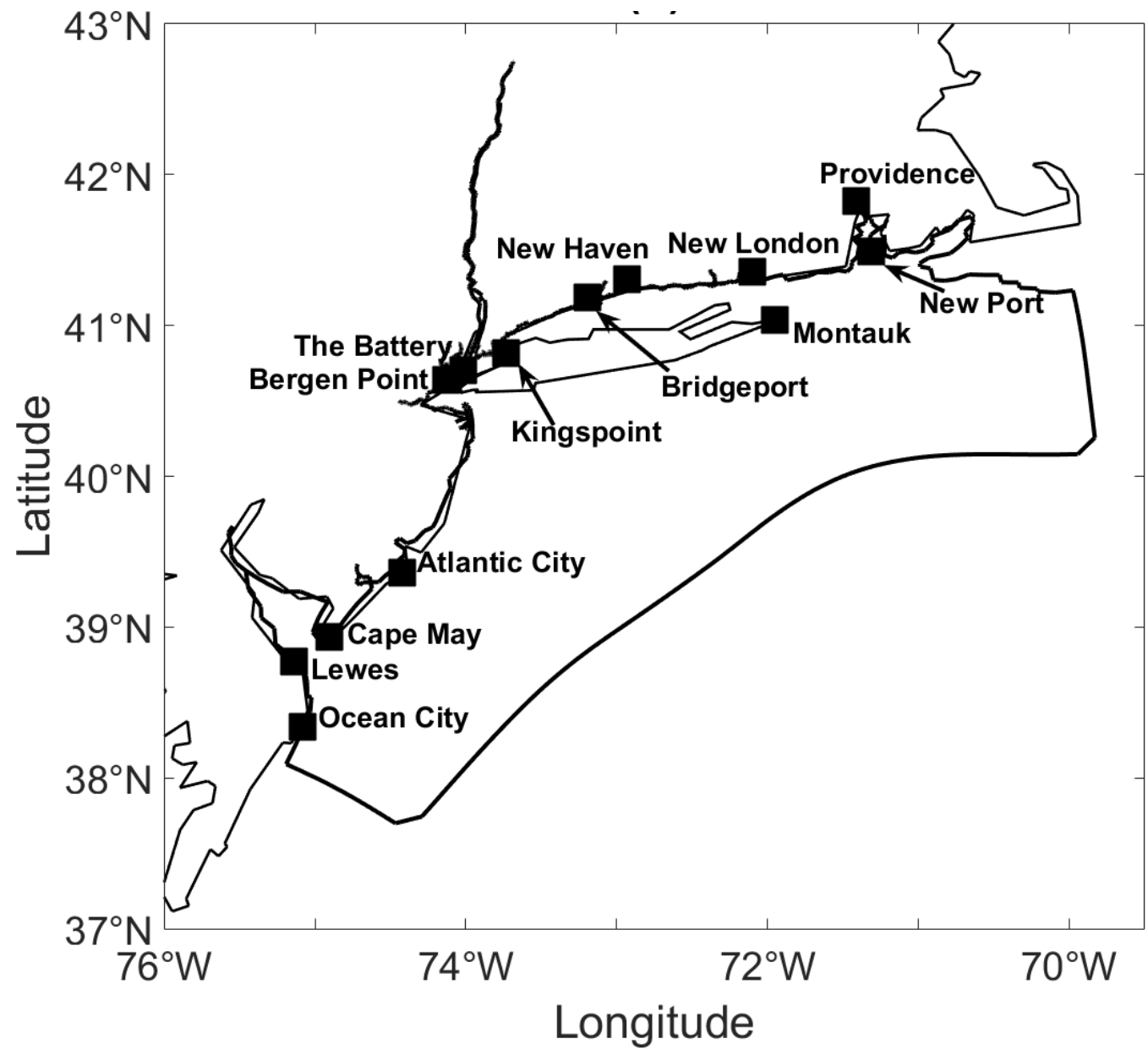
**Compute the Inverse Wavelet Transform**

# Practical Applications to Storm Surge Forecasting

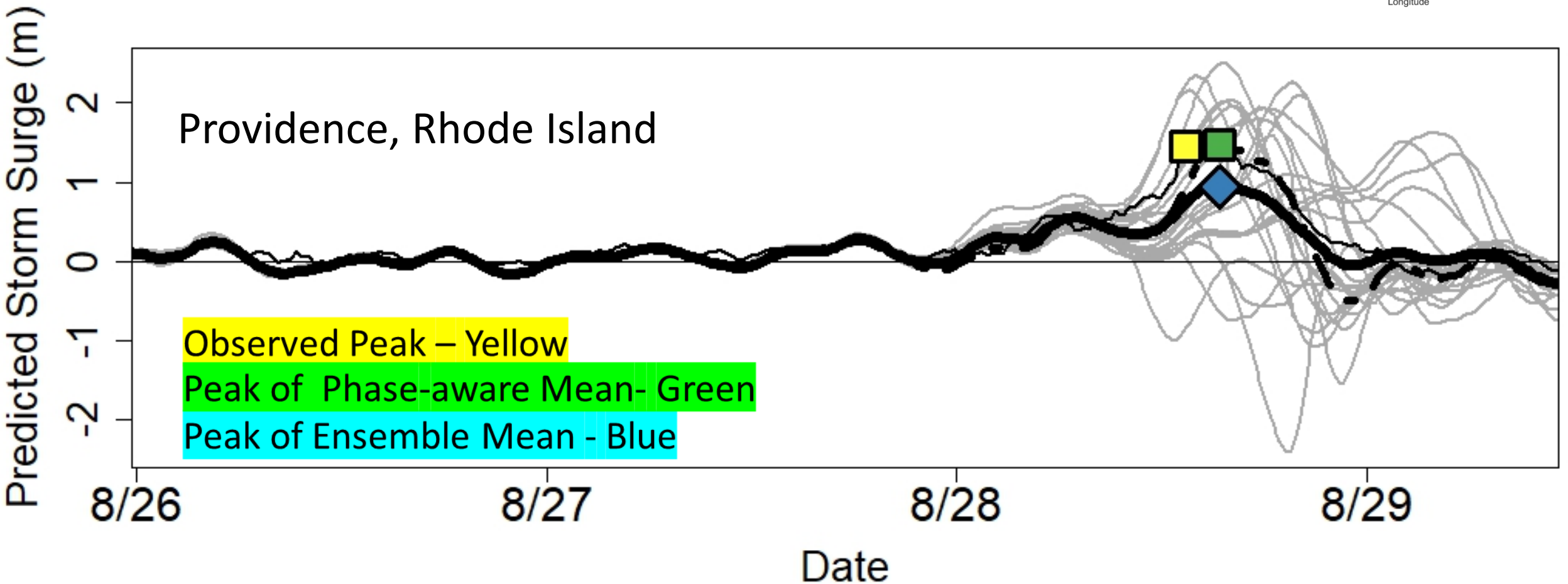
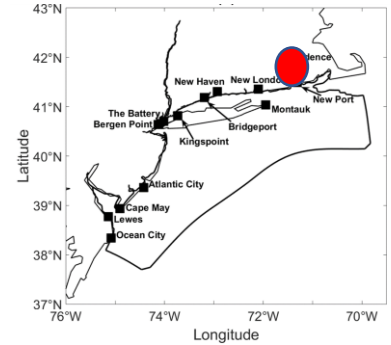
# Storm Surge Forecasting Applications

- Irene and Sandy storm surge forecasts were produced from the New York Harbor Observing and Prediction System (NYHOPS; Georgas, et al., 2016) model.
- The forecasts were issued three days out from the storm events.
- There were 21 ensemble members for each forecast.
- Meteorological forcing was provided from the GEFS Model.
- The performance of the ensemble and phase-aware means were compared across 13 stations.

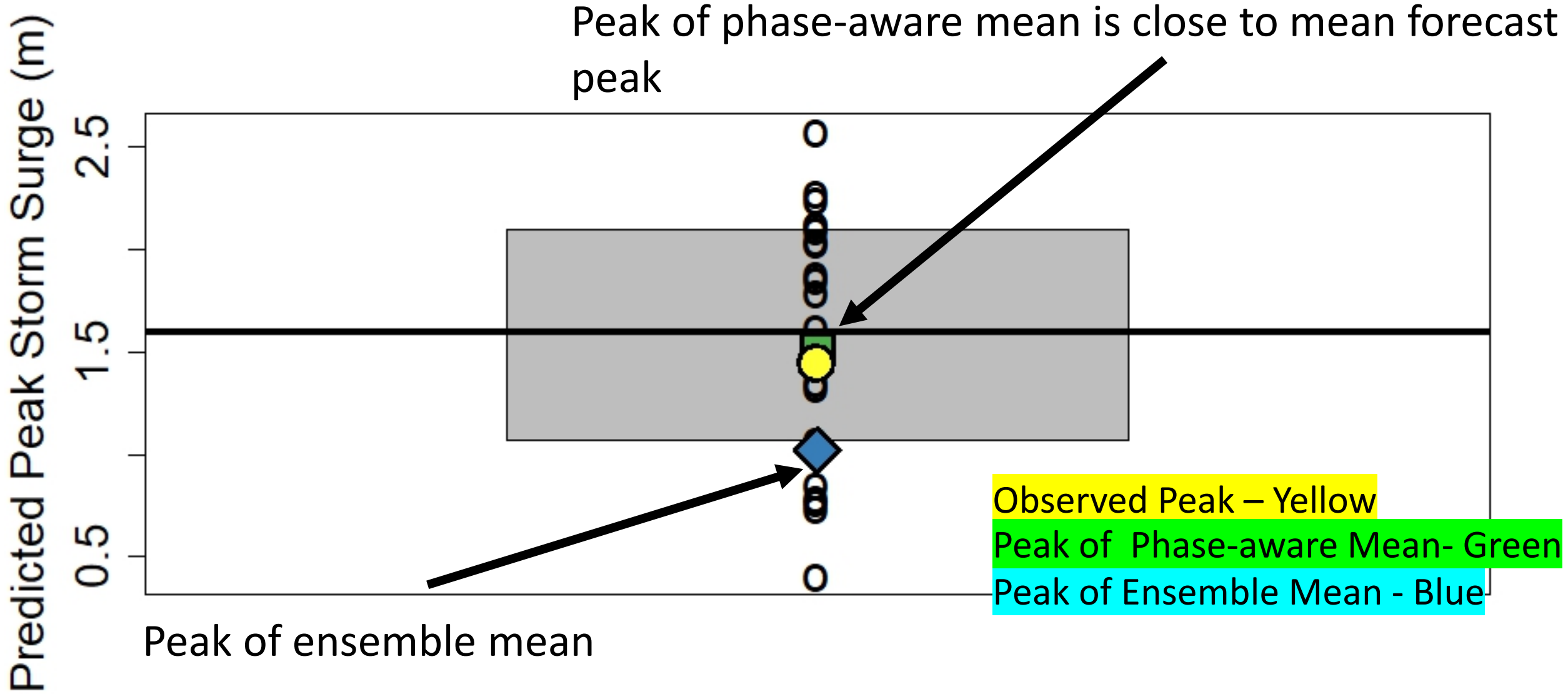




# Hurricane Irene Storm Surge Forecast



# Irene Storm Surge Forecast - Providence

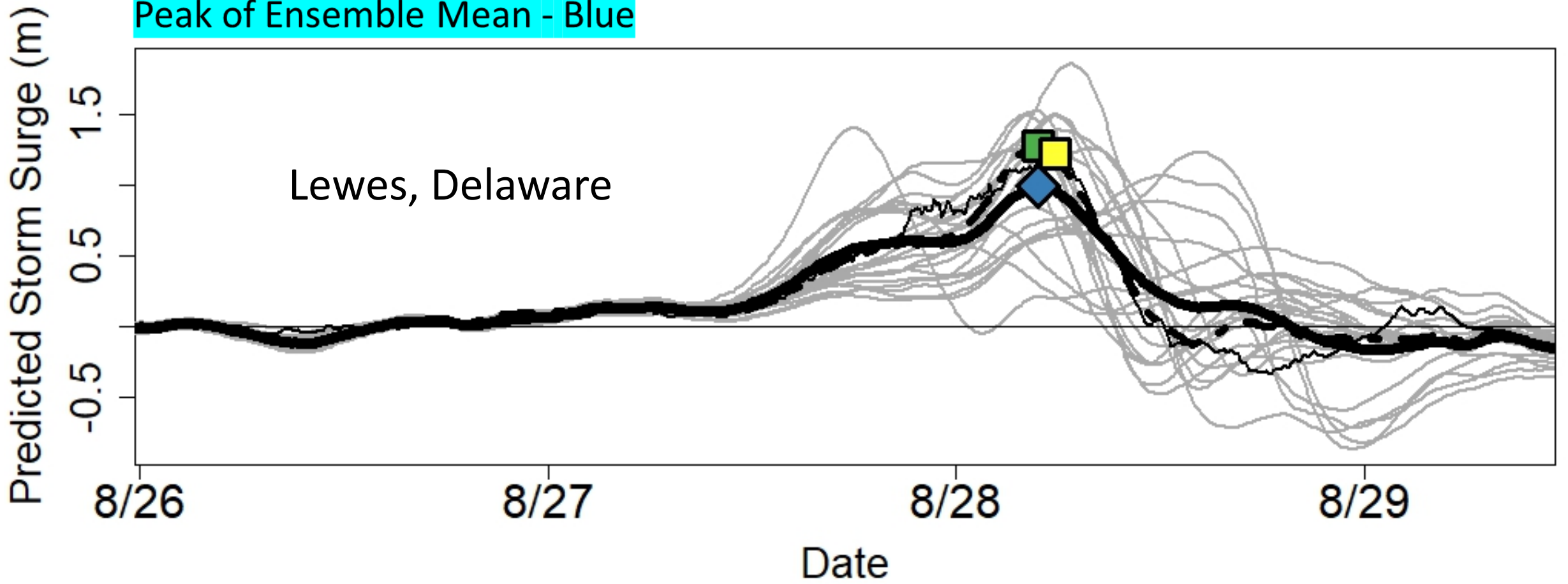
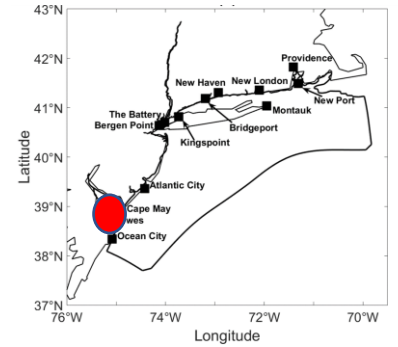


# Hurricane Irene Storm Surge Forecast

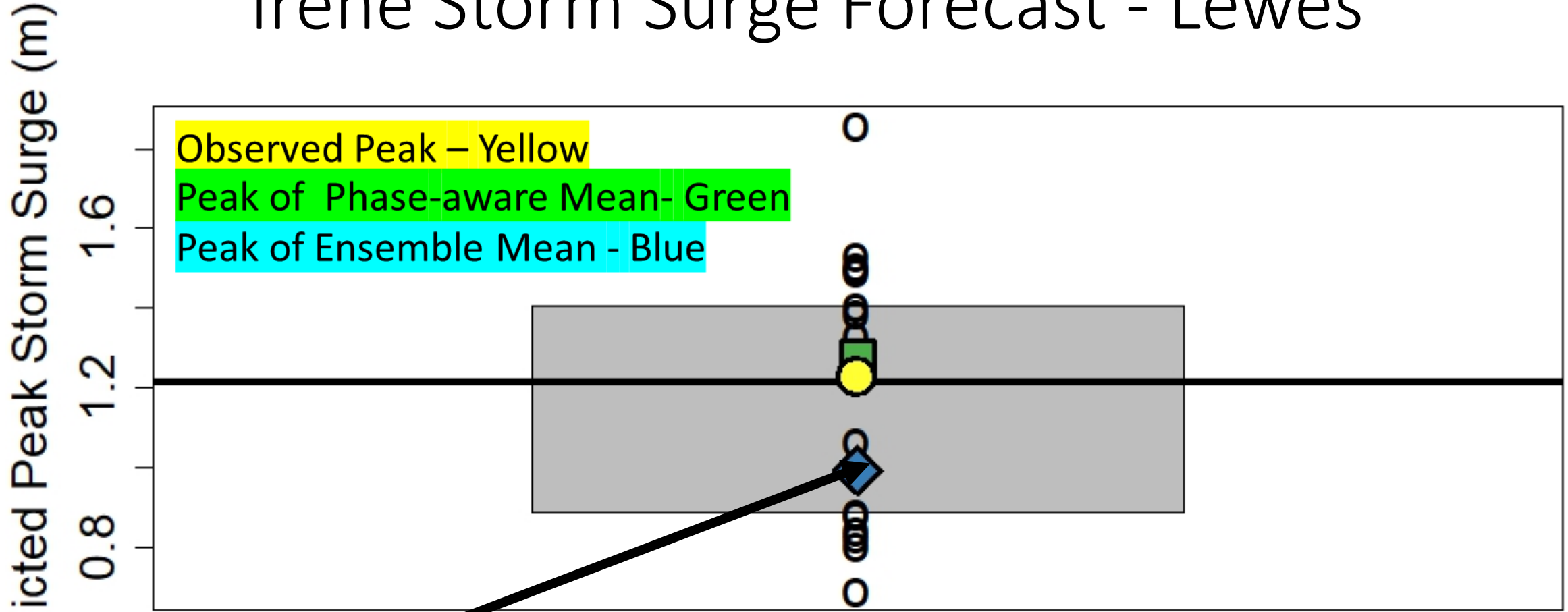
Observed Peak – Yellow

Peak of Phase-aware Mean- Green

Peak of Ensemble Mean - Blue

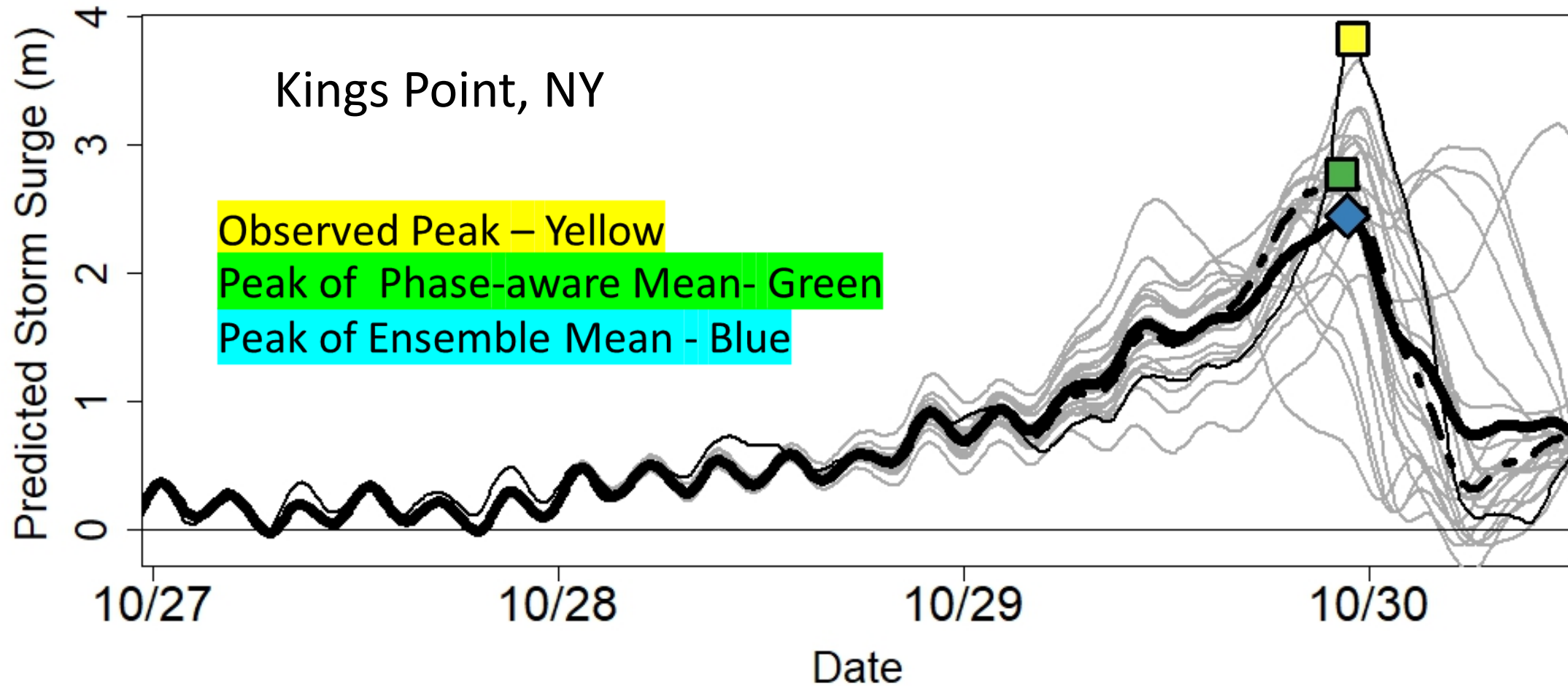
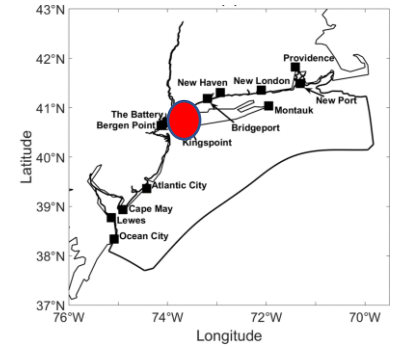


# Irene Storm Surge Forecast - Lewes

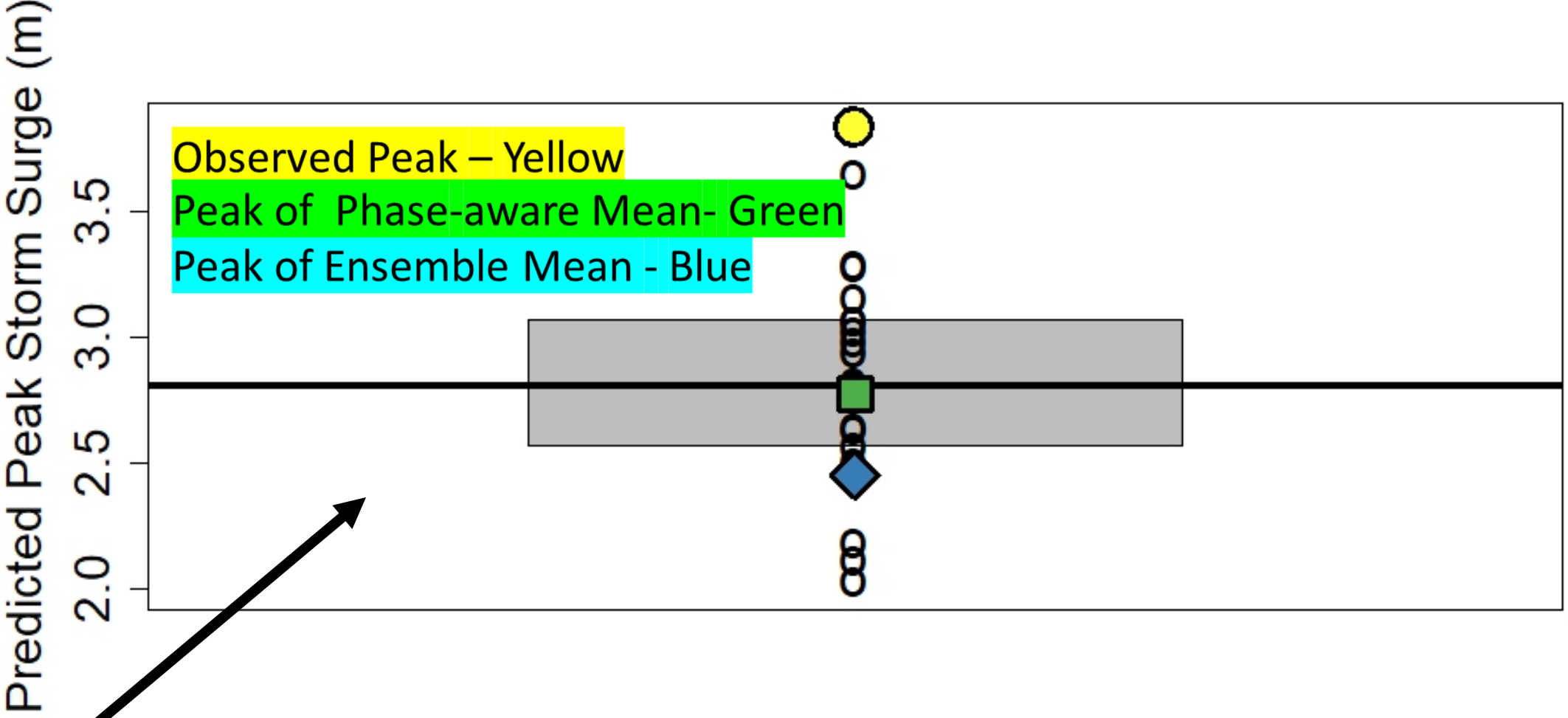


Peak of Ensemble Mean

# Hurricane Sandy Storm Surge Forecast

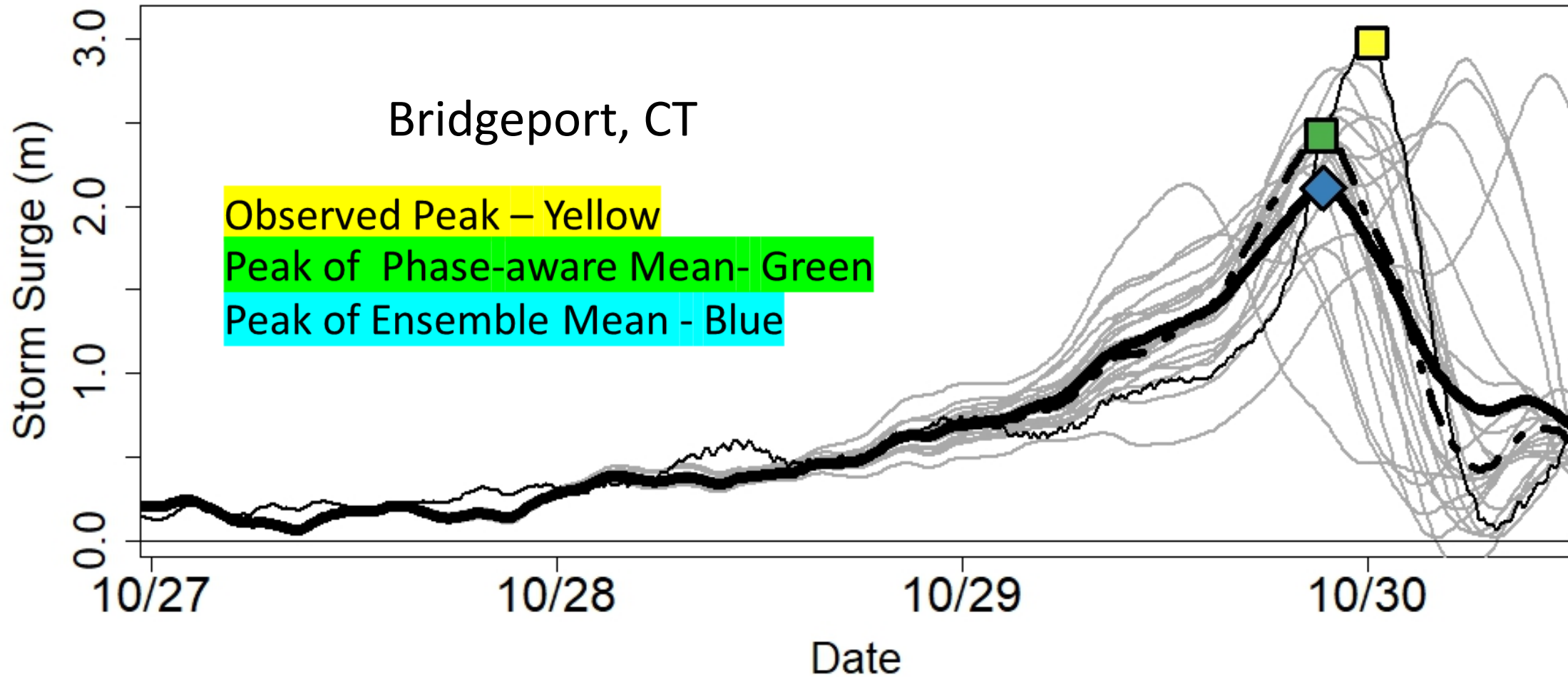
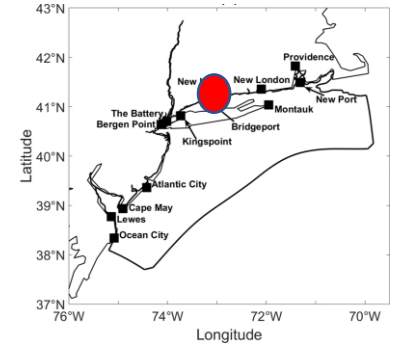


# Hurricane Sandy Storm Surge Forecast – Kings Point



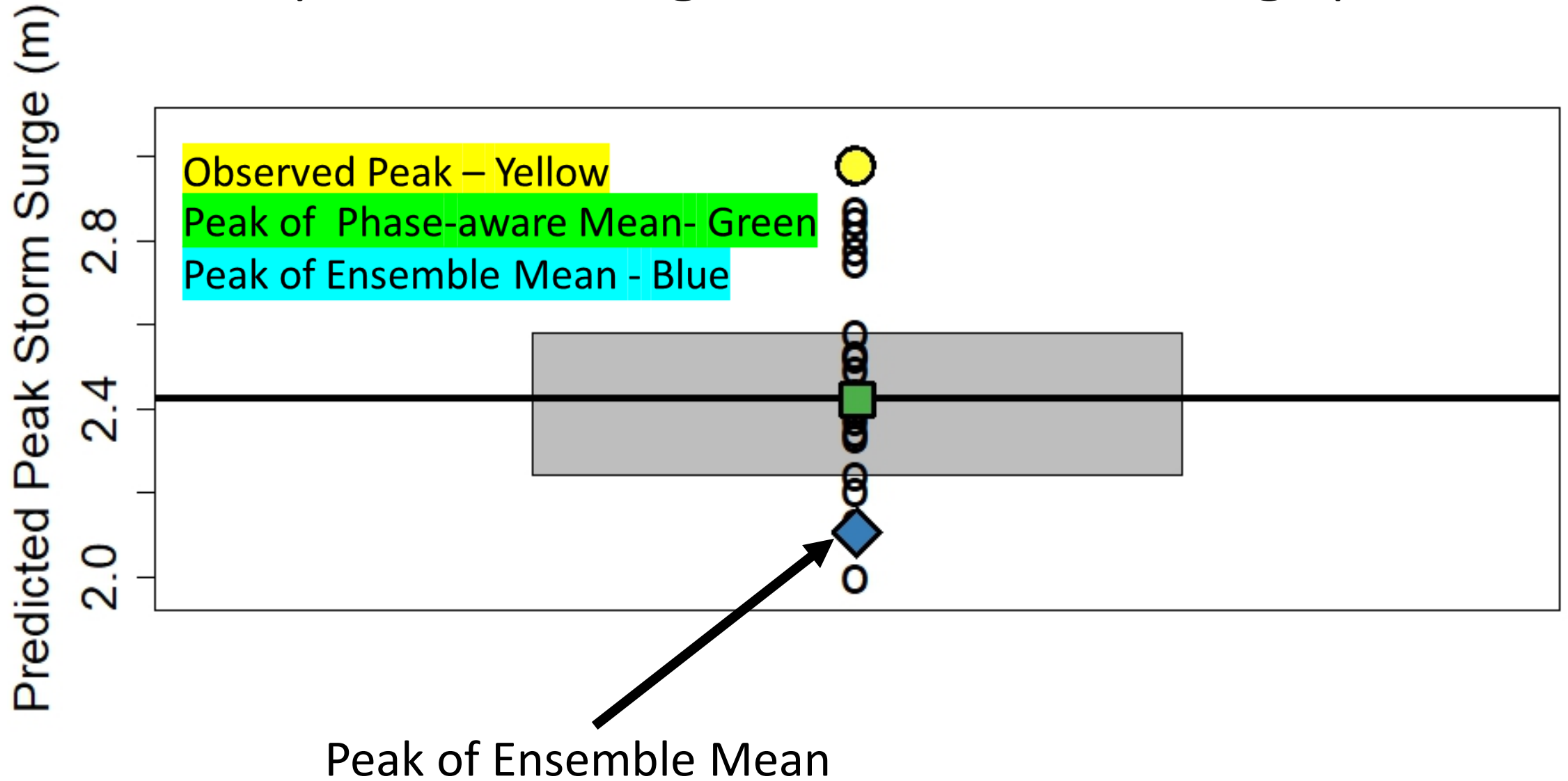
Ensemble Mean Peak

# Hurricane Sandy Storm Surge Forecast

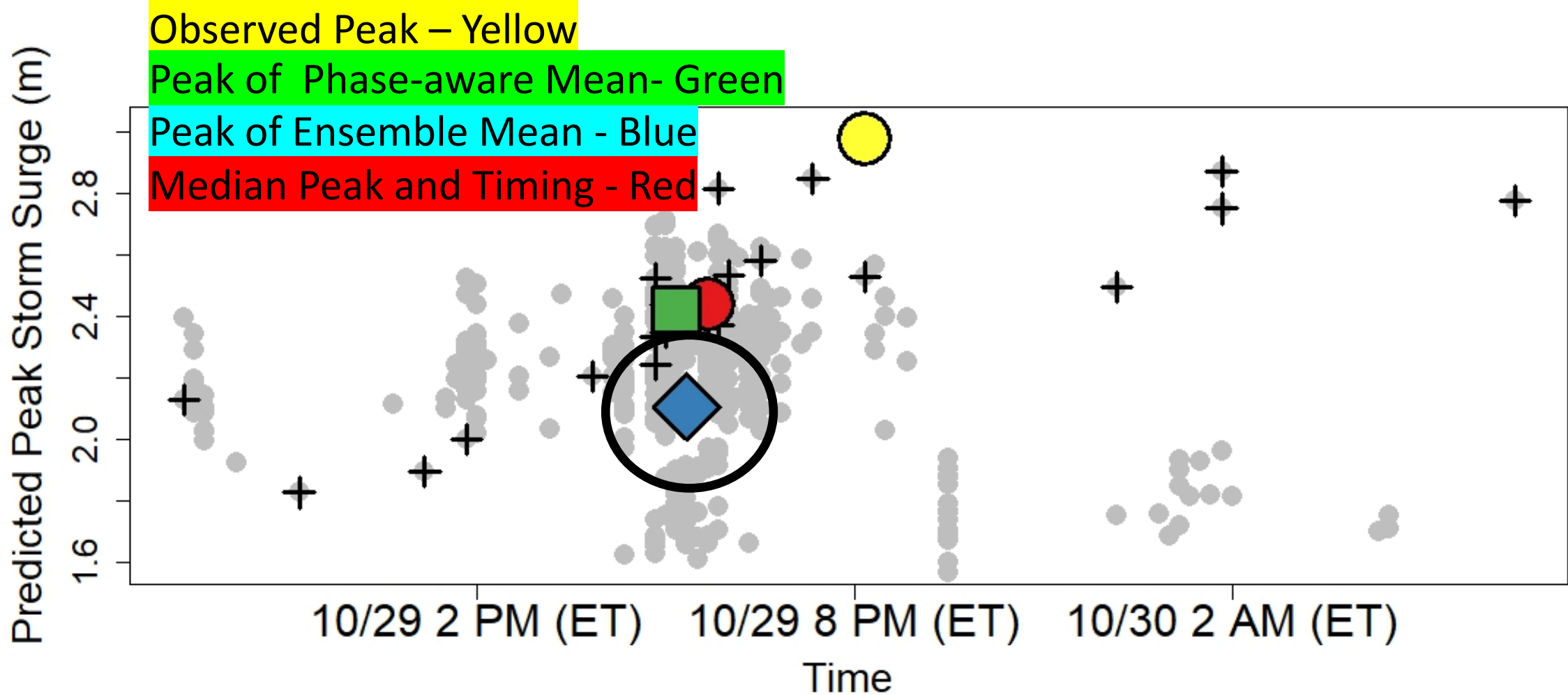




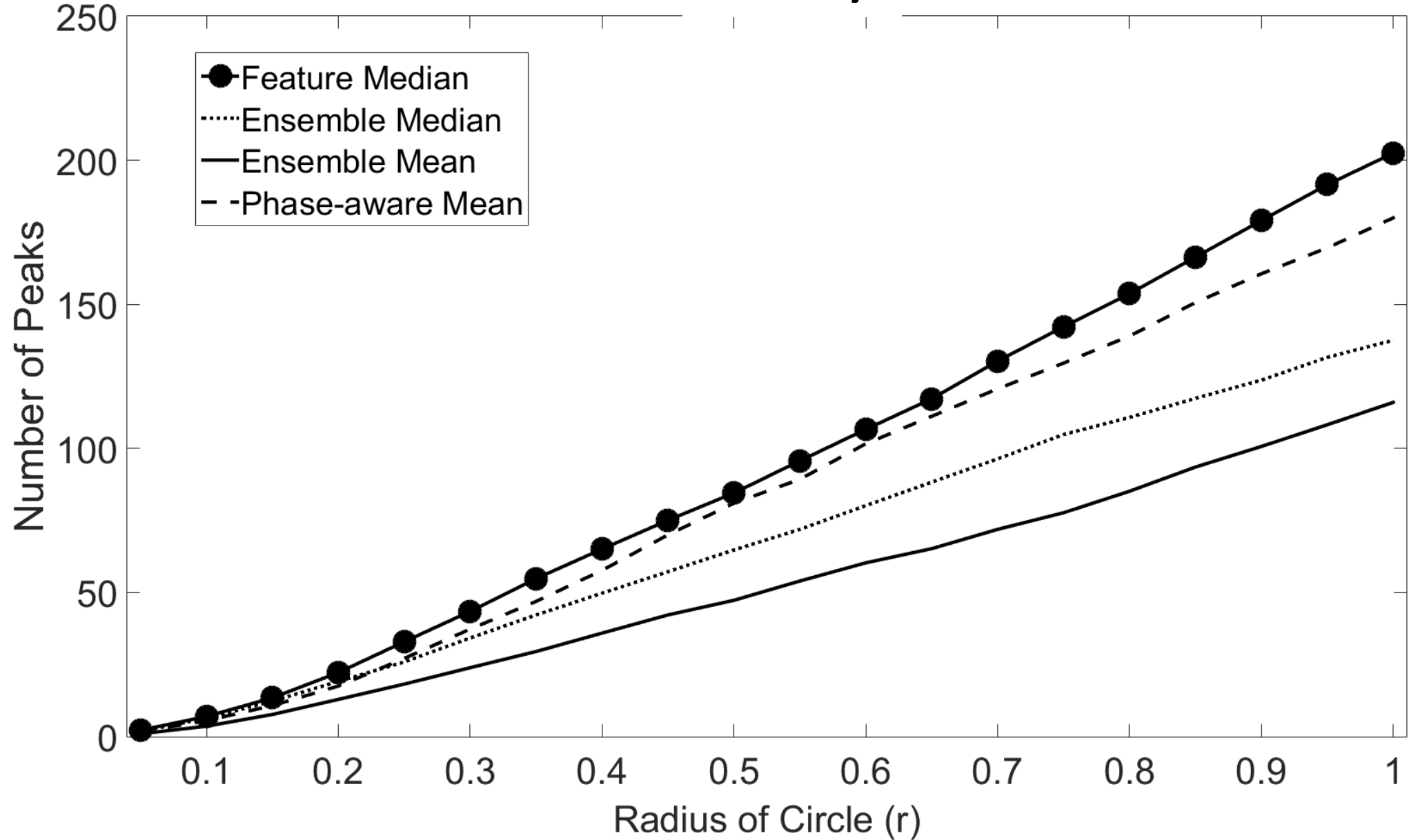
# Sandy Storm Surge Forecast - Bridgeport



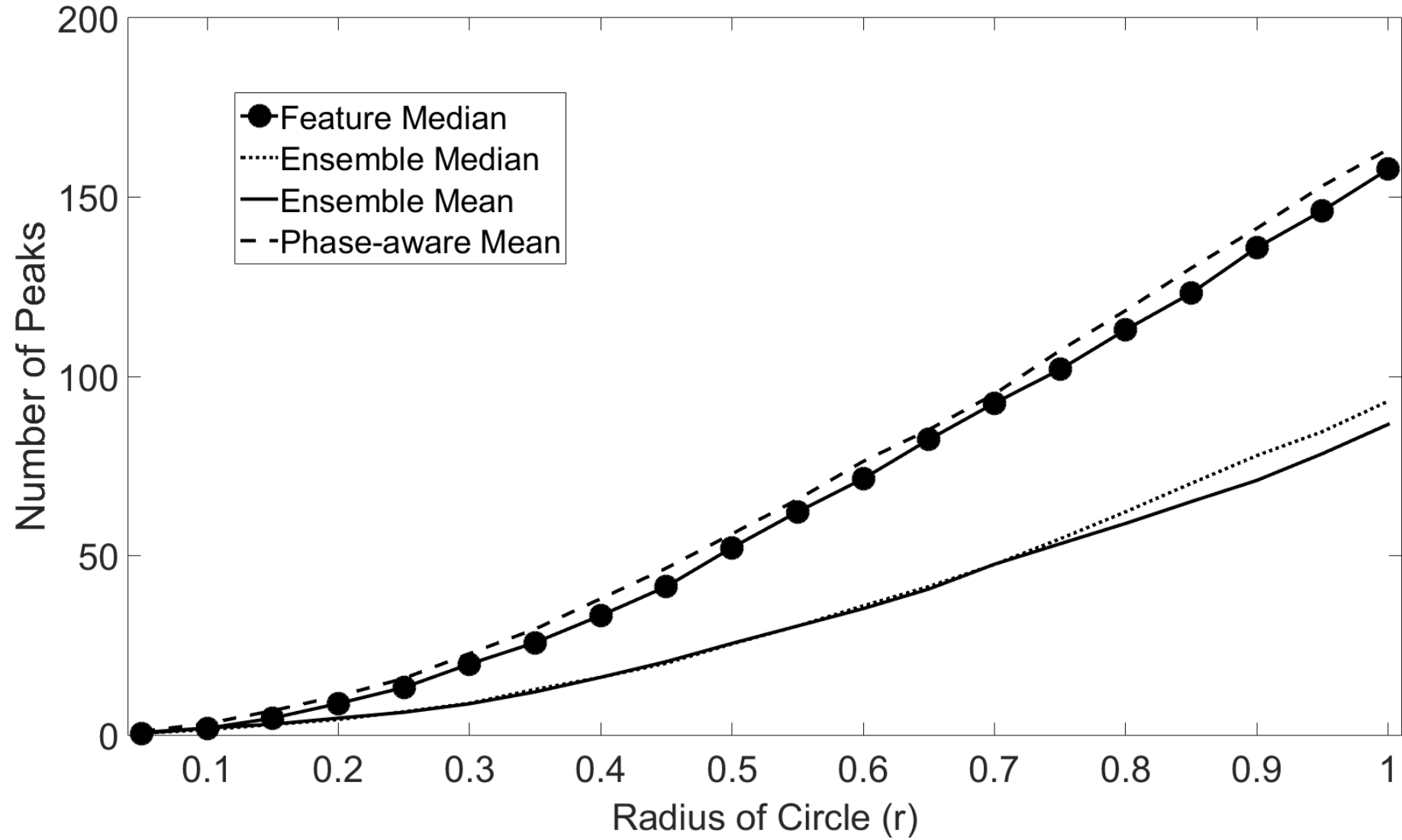
# Sandy Storm Surge Forecast - Bridgeport



# Sandy



# Irene



# Summary

- Timing differences among ensemble members renders the ensemble mean unrepresentative of the ensemble system.
- The amplitude of the ensemble mean can be less than that of any of the individual ensemble members.
- Phase-aware mean remedies several drawbacks of the ensemble mean.
- The number of ensemble members can be increased using a phase-aware extension method.
- Storm surge applications support the results from the theoretical experiments.

# Future Research Directions

- Pseudo-reanalysis data sets
- Monte Carlo methods
- Multi-model ensemble systems
- Composite analyses

# References

- Schulte, J.A and Georgas, N.: Theory and Practice of Phase-aware Ensemble Forecasting, Quarterly Journal of Royal Meteorological Society,144, 2018.
- Georgas, N., Yin, L., Jiang, Y., Wang, Y., Howell, P., Saba, V., Schulte, J A., Orton, P., Wen, B. An Open-Access, Multi-Decadal, Three-Dimensional, Hydrodynamic Hindcast Dataset for the Long Island Sound and New York/New Jersey Harbor Estuaries. J. Mar. Sci. Eng., 4, 48, 2016.

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