Theory and Practice of Phase-aware Ensemble Forecasting

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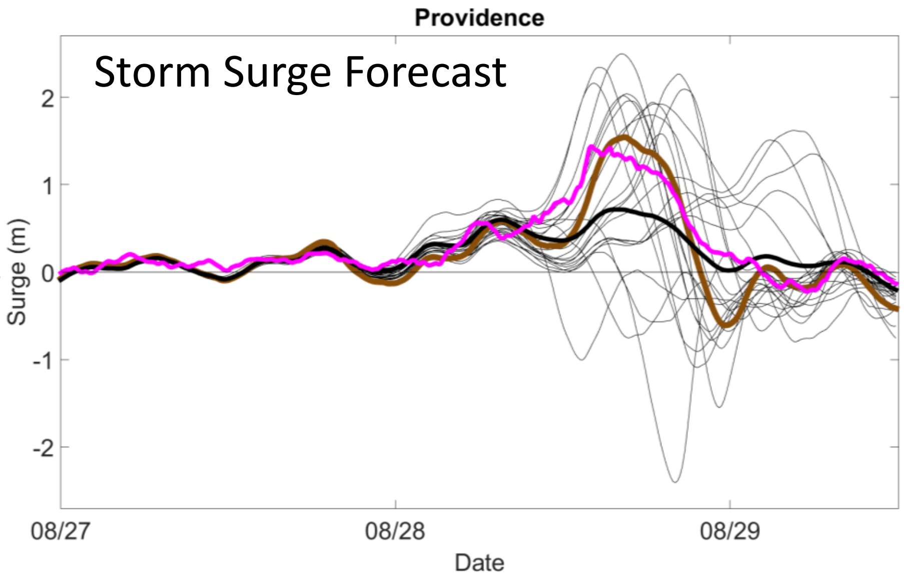
Background

- The ensemble mean is often adopted as the best available estimate of the future behavior.
- The spread around the ensemble mean is related to the uncertainty.
- Two uncertainty sources of interest: timing and intensity of events.
- Timing uncertainty can lead to large root-mean-square errors.
- It is therefore important to develop statistics that operate on timing statistics separately from intensity statistics.



Objectives

1. Introduce a phase-aware mean that provides a more reliable estimate of the most probable outcome than the traditional ensemble mean.

2. Show that one can create at least N^2 statistically sampled ensemble members from an original set of *N* numerical model-derived ones.

Phase-aware Theory

Wavelet Analysis Background

- Let $X_1(t), X_2(t)$ …, $X_N(t)$ be N ensemble members.
- One can compute the wavelet transform of each $X_k(t)$.
- The wavelet transform of each $X_k(t)$ is given by

$$W_k(s,t) = R_k(s\,,t)e^{i\varphi_k(s,t)}$$

The $R_k(s,t) = |W_k(s,t)|$ are related to the intensity of the fluctuations at each Fourier period and time.

The phase (timing) of the ensemble members at each wavelet scale (similar to Fourier period).

Phase-aware Mean Definition

Arithmetic mean modulus (mean intensity of event)

Circular mean phase (mean timing of event)

$$\boldsymbol{Spectral\ phase-aware\ mean} \equiv \widehat{W}(s,t) \equiv \hat{R}e^{i\hat{\varphi}}$$

Inverse wavelet transform converts the spectral mean to a physical time series

$$\widehat{X(t)} = \boldsymbol{phase-aware\ mean\ time\ series}$$

Comparison of Means for Sinusoids

$$X_k(t) = A\sin(Bt + \phi_k)$$

$$\boldsymbol{Ensemble\ mean} = \bar{X}(t, \phi_1, \ldots, \phi_k) = \frac{A}{N}\sum_{k=1}^{N}\sin(Bt + \phi_k)$$

Depends on time and individual phases

$$\boldsymbol{Phase\text{-}aware\ mean} = \hat{X}(t) = A\sin\left(Bt + \hat{\phi}\right)$$

Only depends on time!

Extended (Phase-aware) Ensemble Forecast

$$X_1(t) = A_1 \sin(Bt + \phi_1)$$

$$X_2(t) = A_2 \sin(Bt + \phi_2)$$

$$X_3(t) = A_3 \sin(Bt + \phi_3)$$

Original Ensemble Combinations	Extended Ensemble Combinations		
$\boldsymbol{(A_1, \phi_1)}$	$\boldsymbol{(A_1, \phi_1)}$	(A_1, ϕ_2)	(A_1, ϕ_3)
$\boldsymbol{(A_2, \phi_2)}$	(A_2, ϕ_1)	$\boldsymbol{(A_2, \phi_2)}$	(A_2, ϕ_3)
$\boldsymbol{(A_3, \phi_3)}$	(A_3, ϕ_1)	(A_3, ϕ_2)	$\boldsymbol{(A_3, \phi_3)}$

Ensemble extensions for arbitrary ensemble members

$$W_{ij}(s,t) = R_i(s,t)e^{i\varphi_j(s,t)}$$

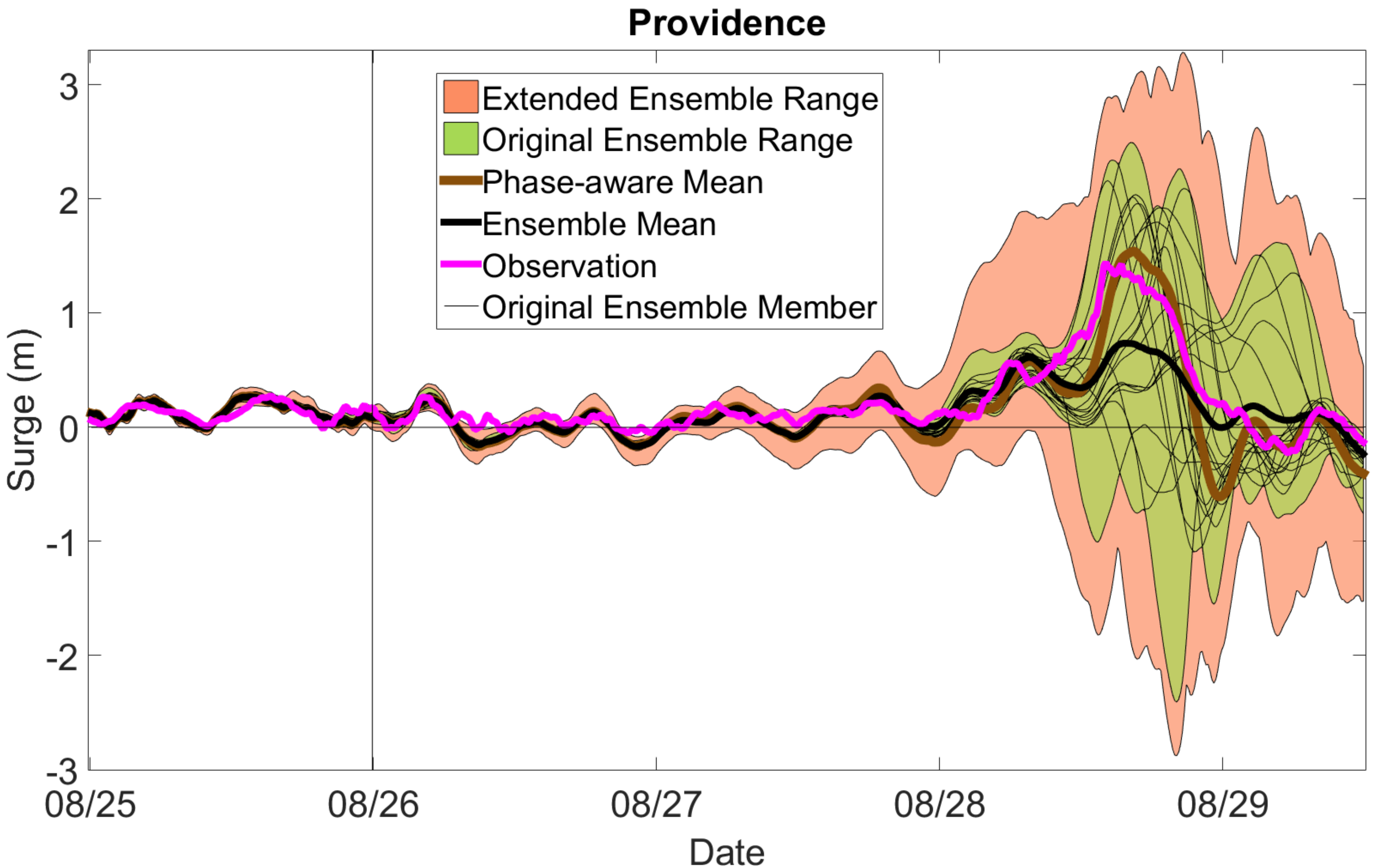
Modulus (intensity) of *i*-th ensemble member (e.g. sinusoid amplitude)

Phase (timing) of *j*-th ensemble member (e.g. sinusoid phase)

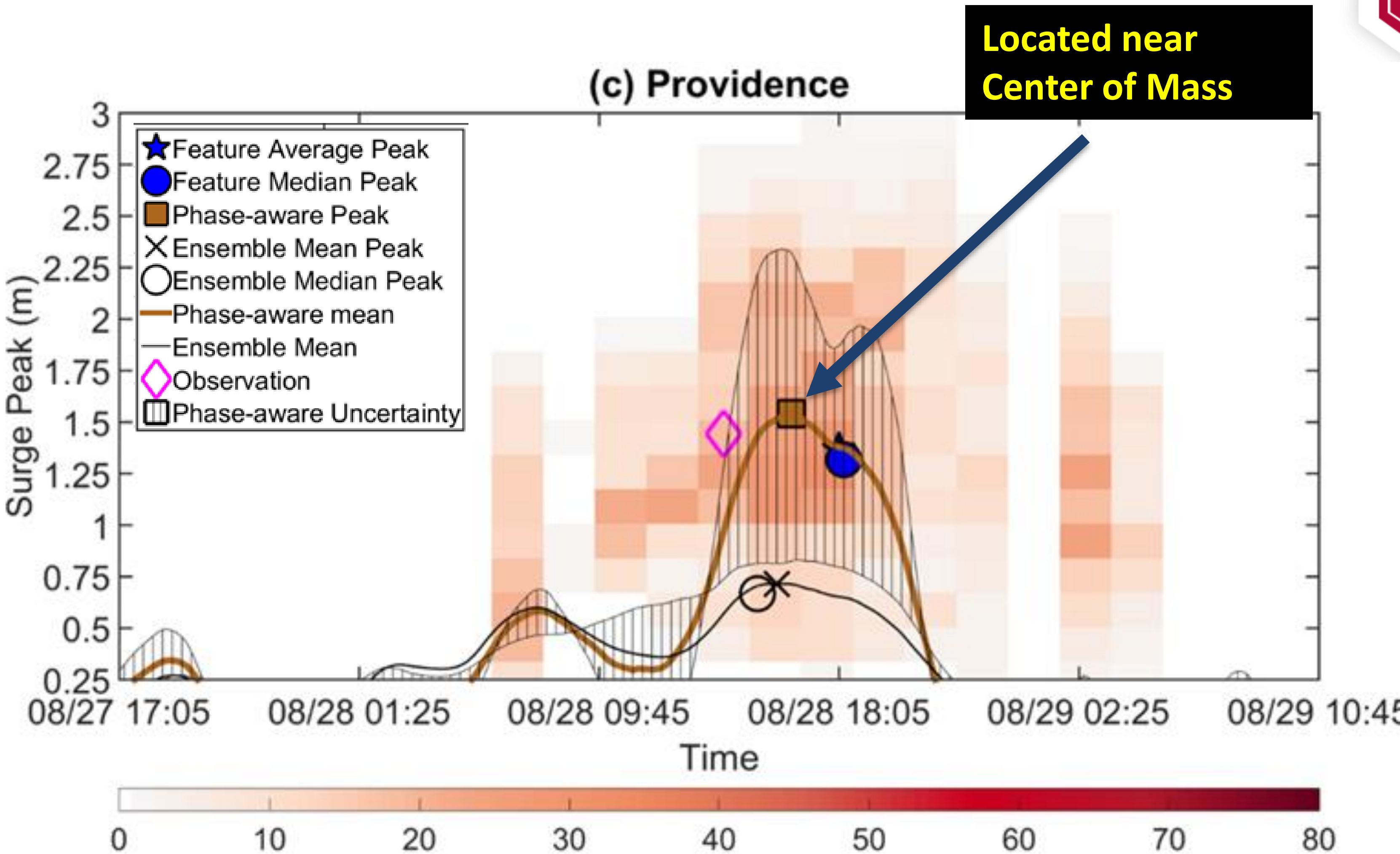
Mix modulus and phase

- This statistical extension allows for extended physical solutions.
- An original ensemble member can predict intensity well but the timing poorly and vice versa.

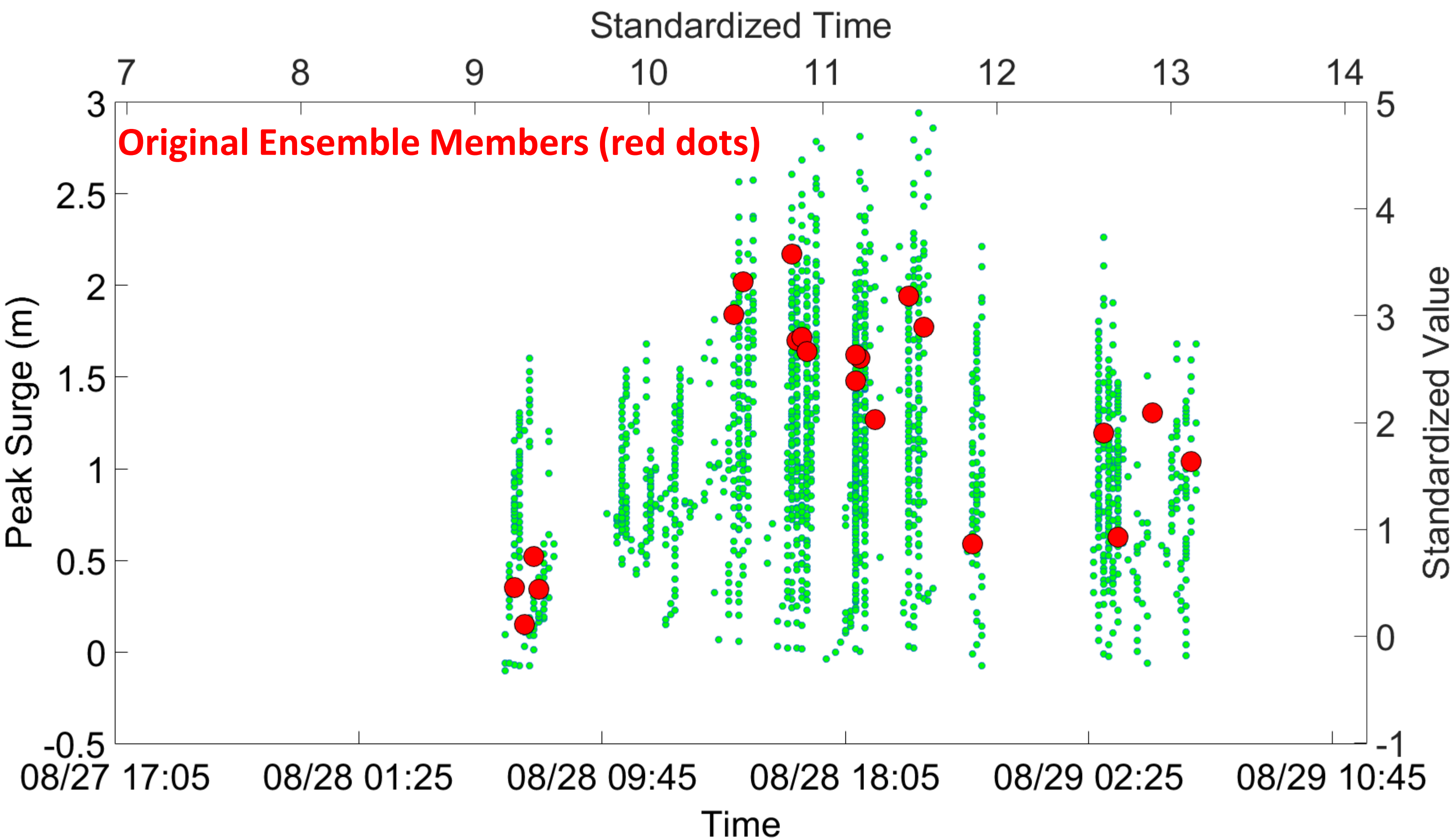
Practical Applications



Providence
Extended Ensemble Range
Original Ensemble Range
Phase-aware Mean
Ensemble Mean
Observation
Original Ensemble Member
Surge (m)
3
2
1
0
-1
-2
-3
08/25
08/26
08/27
08/28
08/29
Date



Located near
Center of Mass
(c) Providence
Feature Average Peak
Feature Median Peak
Phase-aware Peak
Ensemble Mean Peak
Ensemble Median Peak
Phase-aware mean
Ensemble Mean
Observation
Phase-aware Uncertainty
Surge Peak (m)
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
08/27 17:05
08/28 01:25
08/28 09:45
08/28 18:05
08/29 02:25
08/29 10:45
Time
0
10
20
30
40
50
60
70
80



Standardized Time
7
8
9
10
11
12
13
14
Original Ensemble Members (red dots)
3
2.5
2
1.5
1
0.5
0
-0.5
Peak Surge (m)
5
4
3
2
1
0
-1
Standardized Value
08/27 17:05
08/28 01:25
08/28 09:45
08/28 18:05
08/29 02:25
08/29 10:45
Time

Conclusions

- A phase-aware ensemble mean remedies the problem that the traditional ensemble mean is an underestimation of the most probable peak surge (**Objective 1**).
- The phase-aware mean falls more closely to the center of masses of joint distributions.
- Using wavelet analysis, one can create at least N^2 statistically sampled ensemble members from an original set of N numerical model-derived ones (**Objective 2**).
- The result of creating N^2 ensemble members is a better-dispersed ensemble obtained without having to run more computationally expensive numerical models.

Information

- The phase-aware methodology is currently running operationally (http://hudson.dl.stevens-tech.edu/sfas/).
- A software package has been developed.
- A paper was submitted to the Quarterly Journal of the Royal Meteorological Society.
- If interested in the paper or software, please send email to jschulte@steven.edu or ngeorgas@stevens.edu.