Cumulative areawise testing in wavelet analysis and its application to the Pacific Decadal Oscillation

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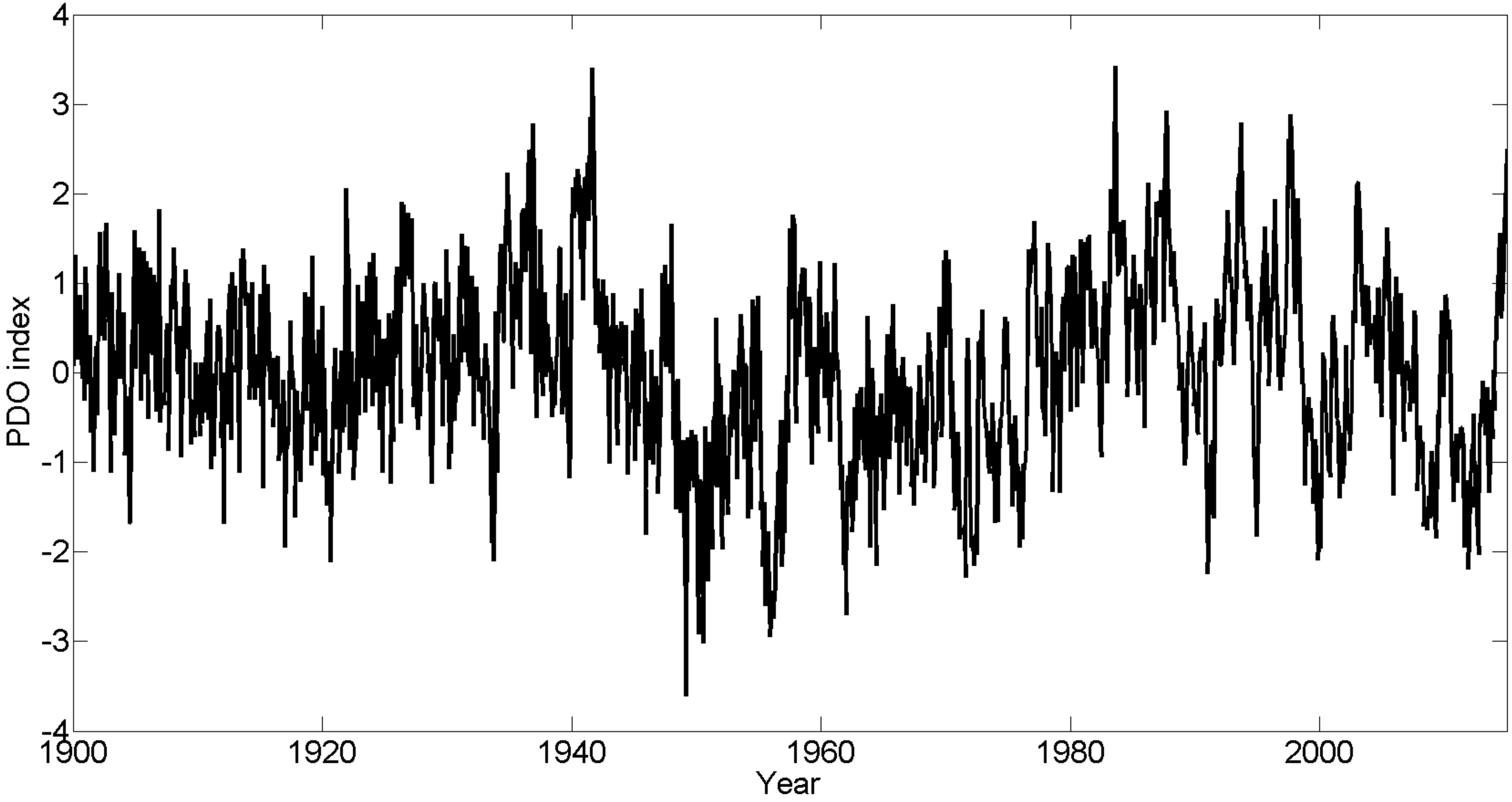
PDO Index - Definition

- The leading principle component of monthly SST anomalies in the North Pacific Ocean, poleward of 20°N.
- The monthly mean global average SST anomalies are removed to separate this pattern of variability from any "global warming" signal

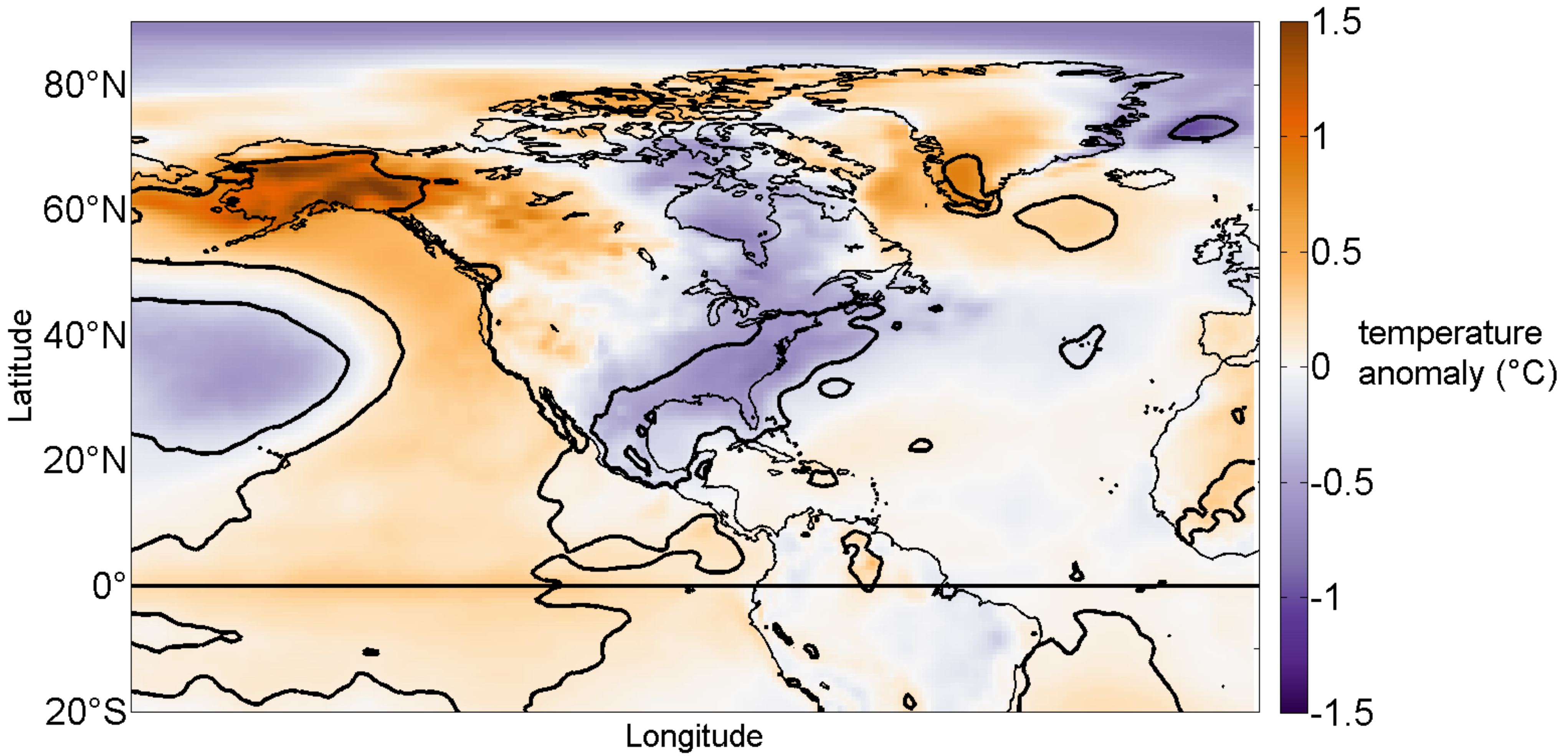
PDO index



PDO Index – Oscillation or Noise?

- Some studies suggest that the PDO is the reddened response to atmospheric white forcing and the ENSO signal (Newmann et al., 2003)
- Wavelet analysis will allow us to provide statistical evidence in favor of one the hypotheses

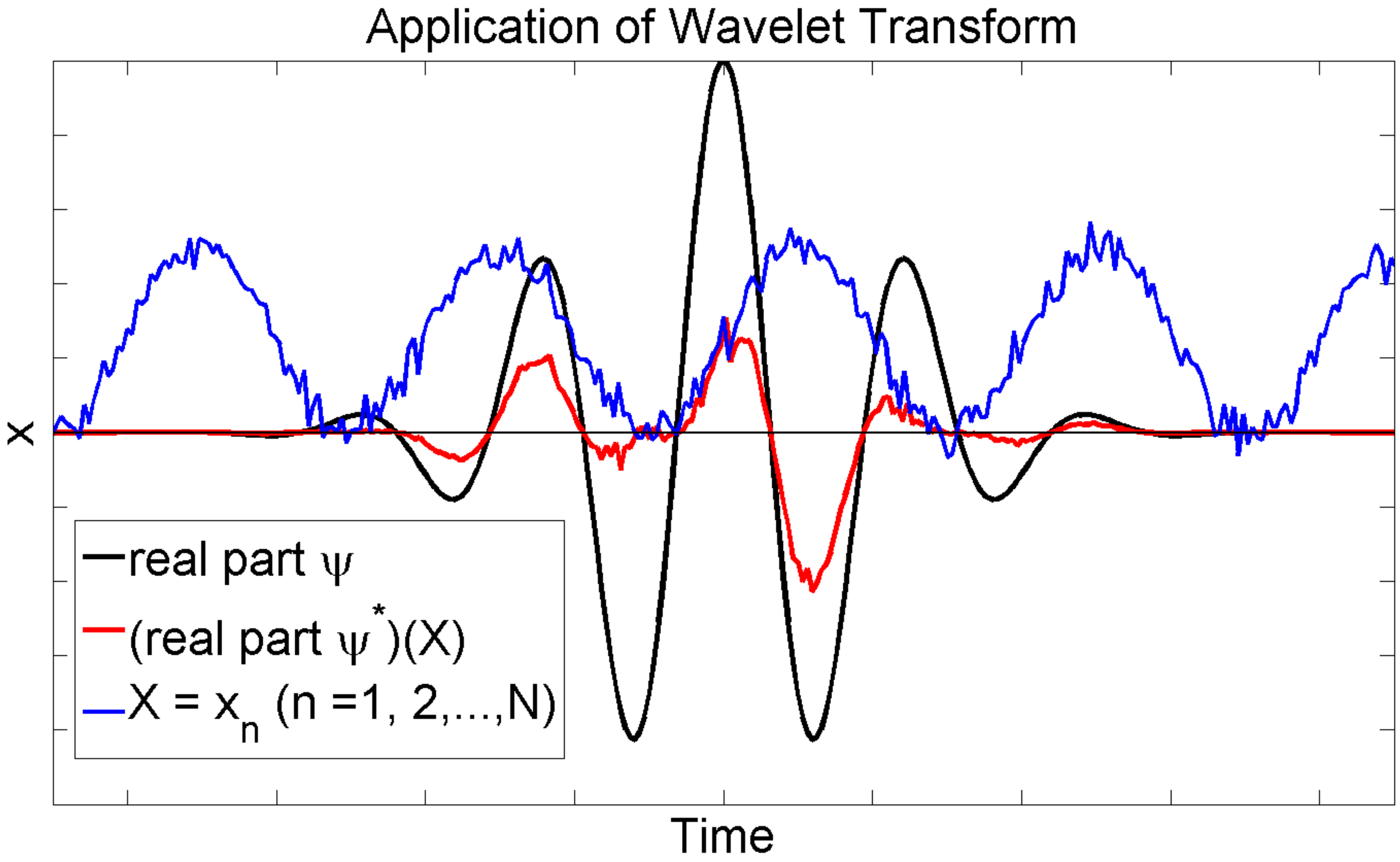
Positive PDO Index and DJF temperature



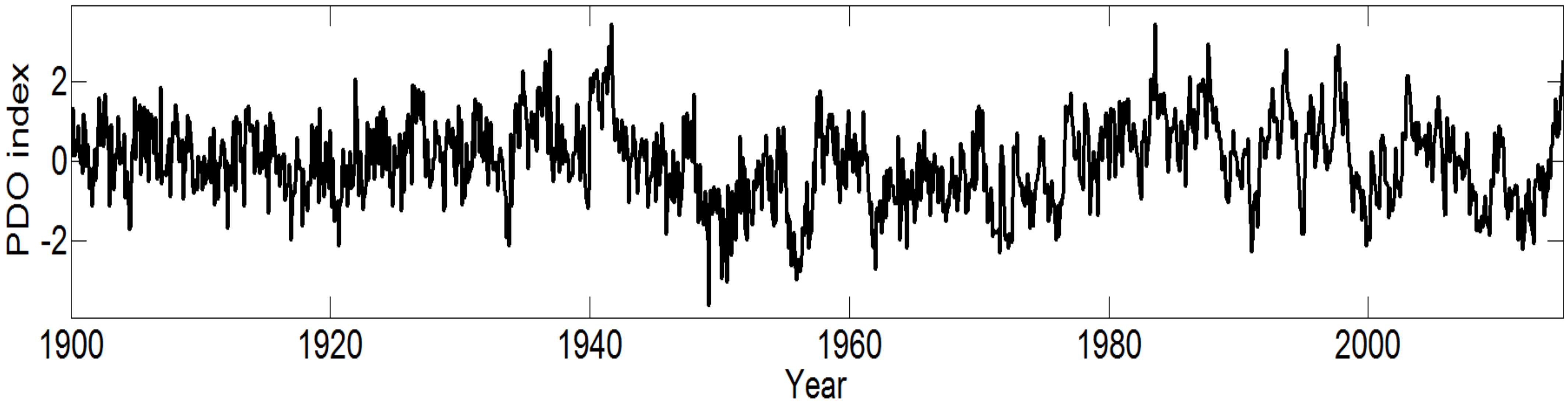
The Wavelet Transform

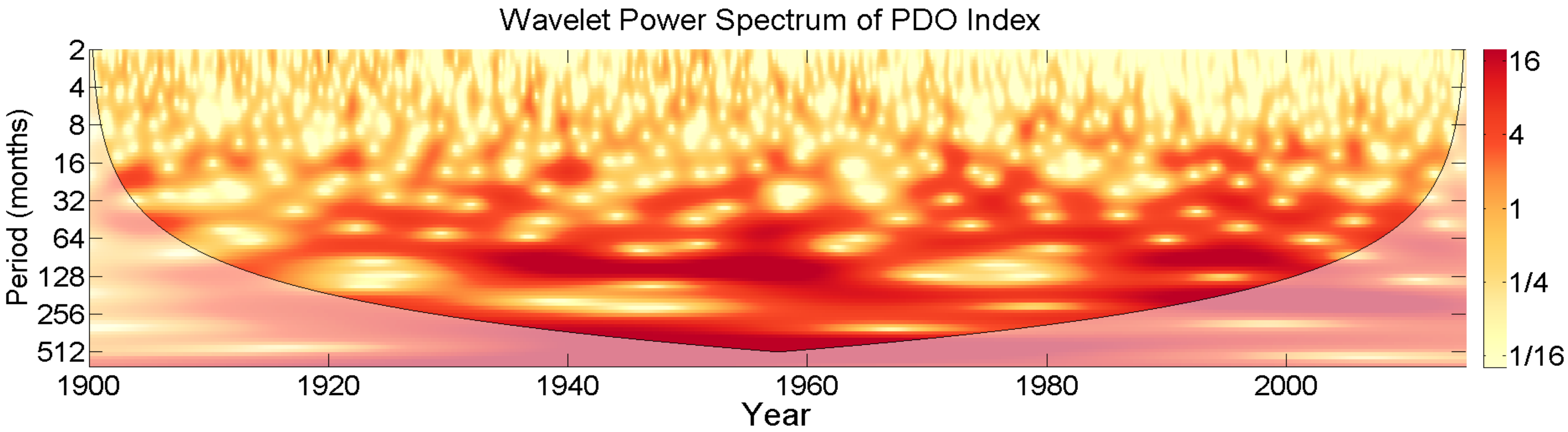
- $W_n^X(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^{N} x_{n'} \psi_0[(n' - n)\frac{\delta t}{s}]$
- $x_{n'}$ = time series
- ψ_0 = Morlet wavelet
- δt = time step determined from data
- $\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}$,
- Wavelet Power = $\left(W_n^X(s)\right)^2$

The Wavelet Transform



Wavelet Power Spectrum of PDO Index





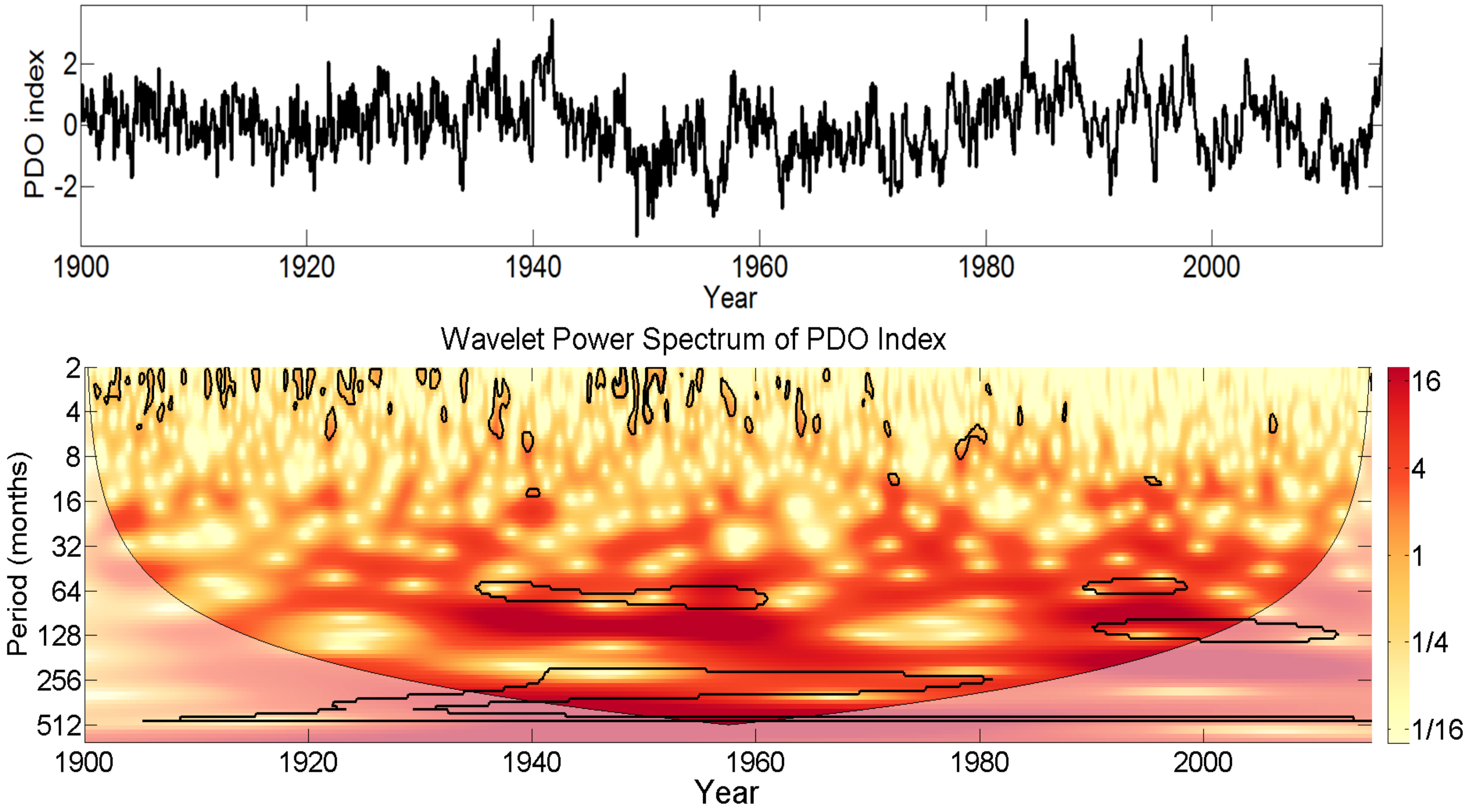
Existing Significance Testing Procedures

- Pointwise significance testing (Torrence and Compo, 1998)
- Areawise significance testing (Maruan et al., 2004)
- Geometric significance testing and topological methods (Schulte et al., 2015)

Pointwise Significance Testing

- Assigns to each wavelet power coefficient a *p*-value, which is the probability of obtaining a test statistic as extreme as the observed value calculated when the null hypothesis is true.
- A typical noise hypothesis used in climate science is red noise

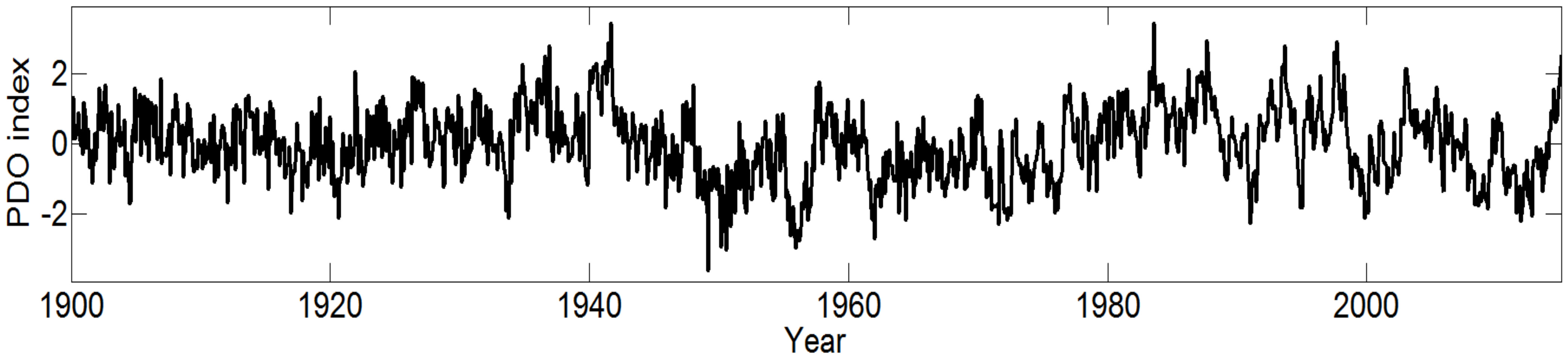
Pointwise Significance Testing

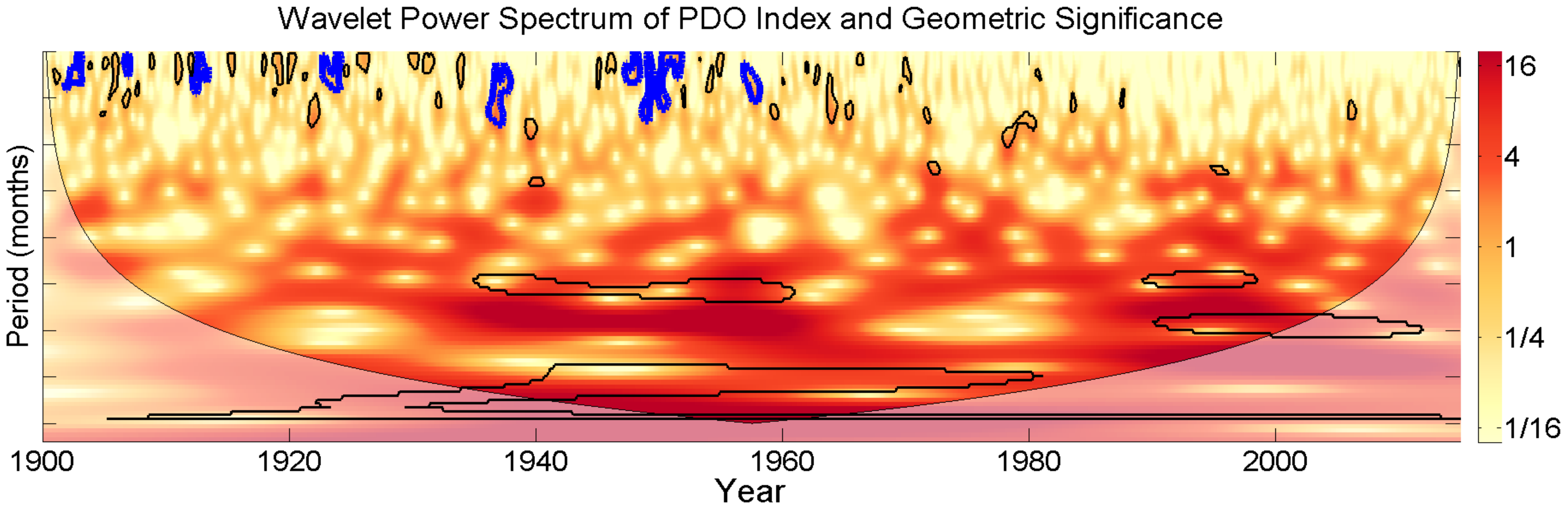


Geometric Significance Testing

- Assesses the significance of patches based on their area.
- Test statistic is normalized area
- $\text{Normalized area} = \dfrac{\text{area of patch}}{\text{scale}-\text{coordinate of centroid squared}}$

Wavelet Power and Geometric Significance





Pitfalls and Strengths

×	Computationally Efficient	Statistical Power	Free of Binary Decision
Pointwise	✓	✗	✓
Areawise	✗	Okay	✗
Geometric	✓	Okay	✗
Cumulative Areawise	✓	✓	✓

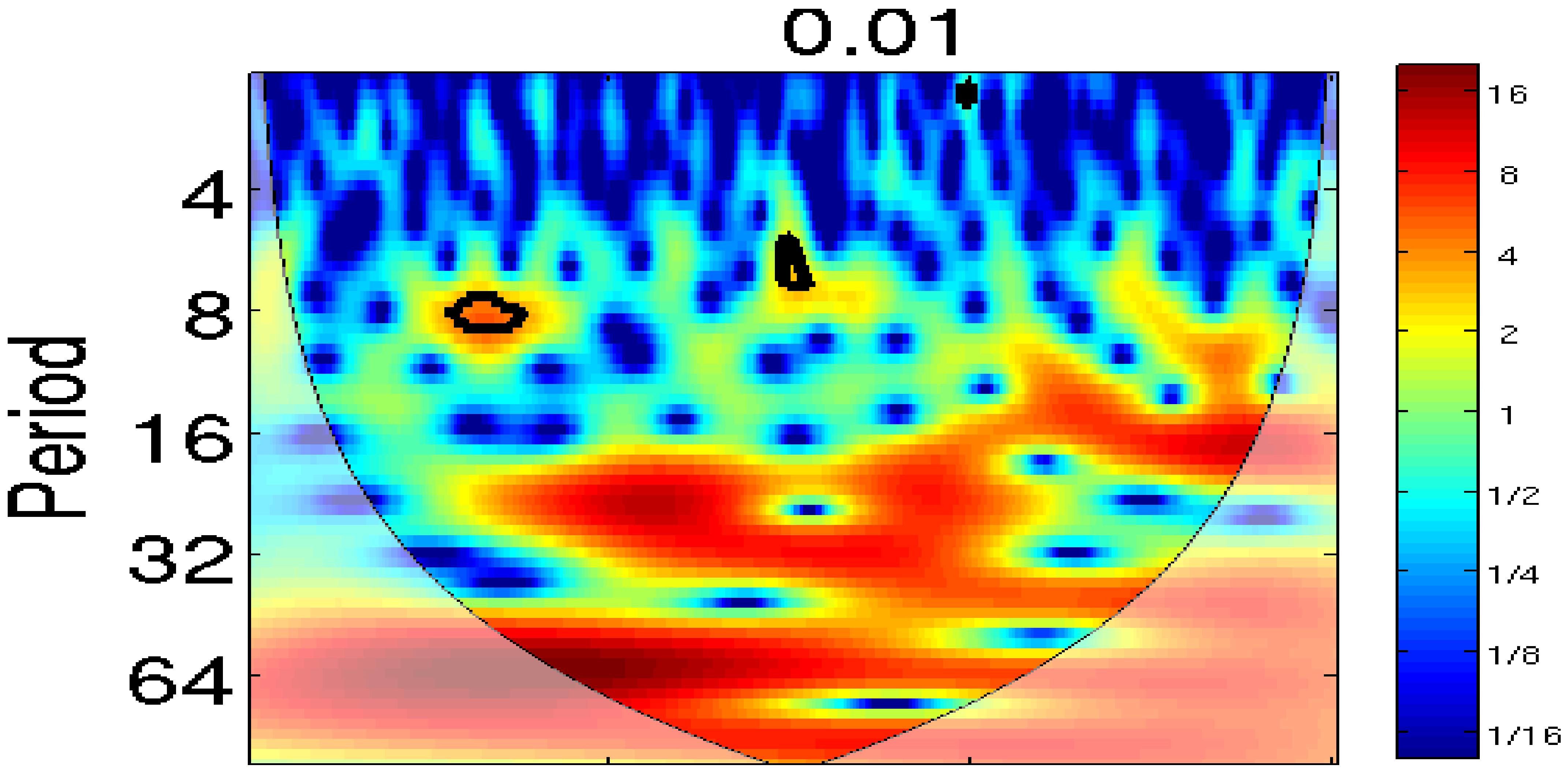
Cumulative Areawise Testing

Topology

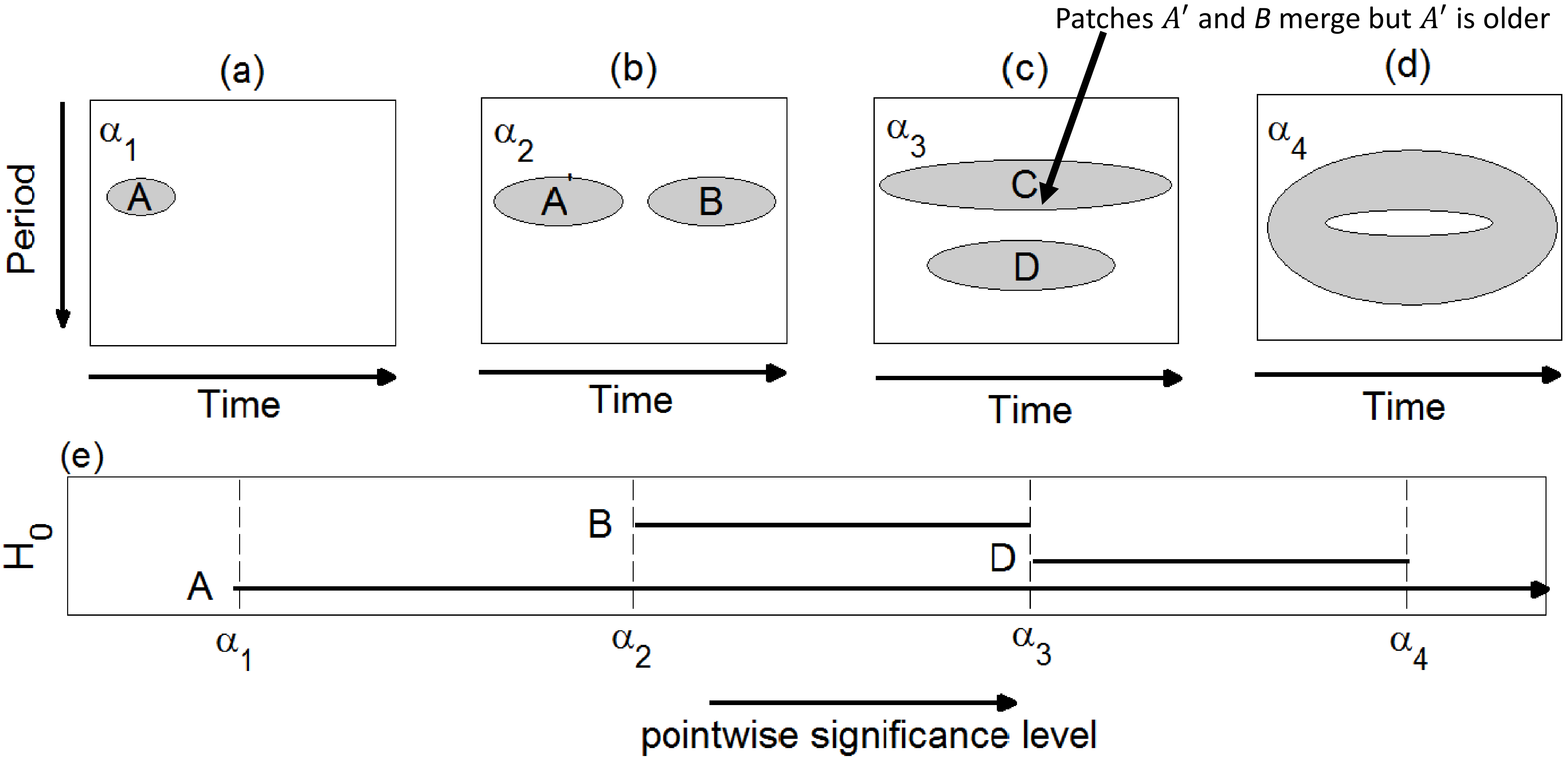
Cumulative Areawise Testing

- Assesses the significance of wavelet power coefficients by understanding how the normalized areas of patches change under a changing pointwise significance level.
- It is therefore important to understand how patches change as one changes the pointwise significance level
- The test statistic in this case will be the cumulative sum of areas over all pointwise significance levels
- The integrated quantity will remove the binary decision from which the geometric test suffers.

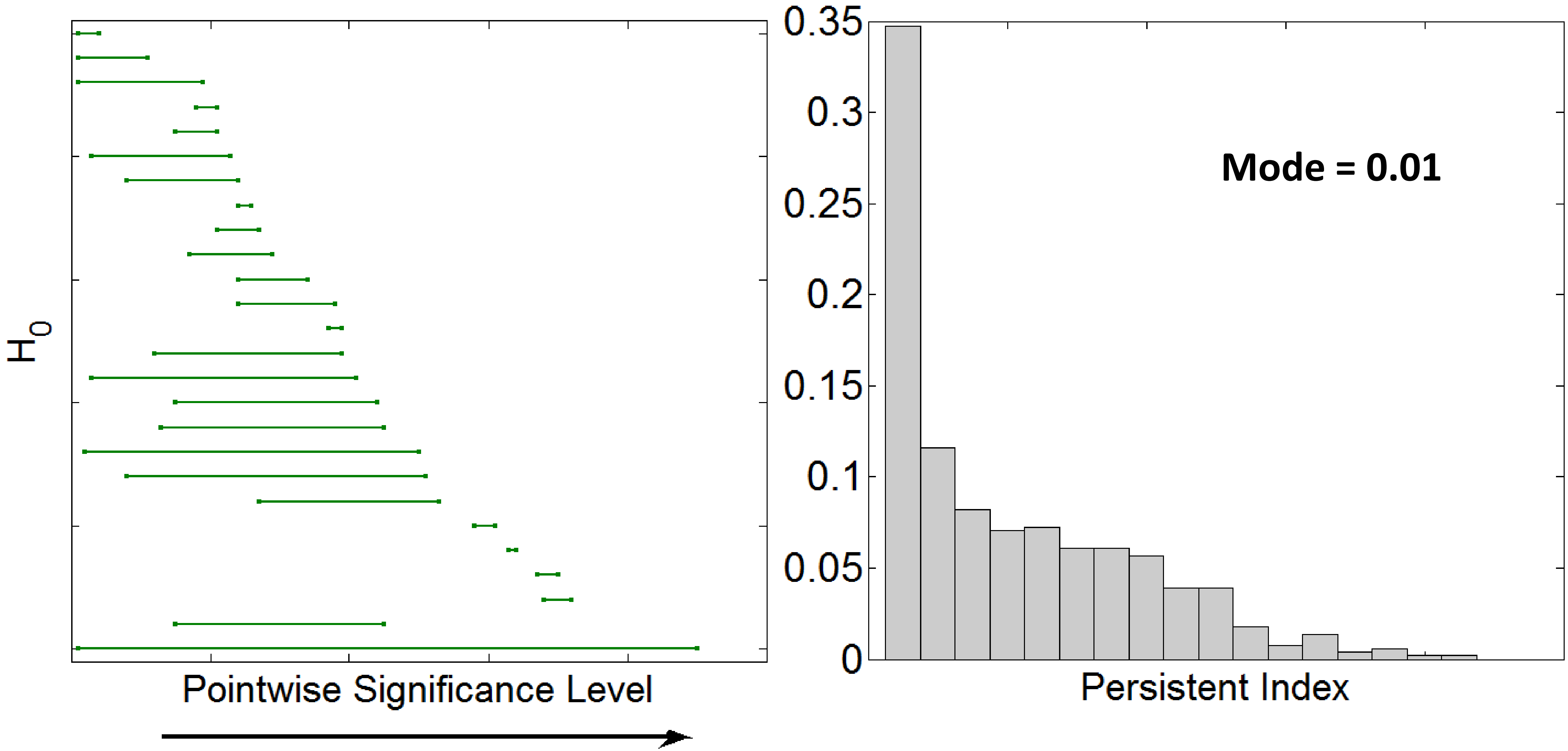
Persistent Homology



Persistent Homology - Barcodes



Barcodes and Persistent Indices for Patches



Cumulative Areawise Testing

Development

Definition of Geometric Pathway

- A **geometric pathway** will be defined as a collection $\mathcal{P}$ of *r* patches at the corresponding pointwise significance levels α_1,…,α_r such that

$$P_1 \subseteq P_2 \subseteq P_3 \subseteq \cdots \subseteq P_r$$

and

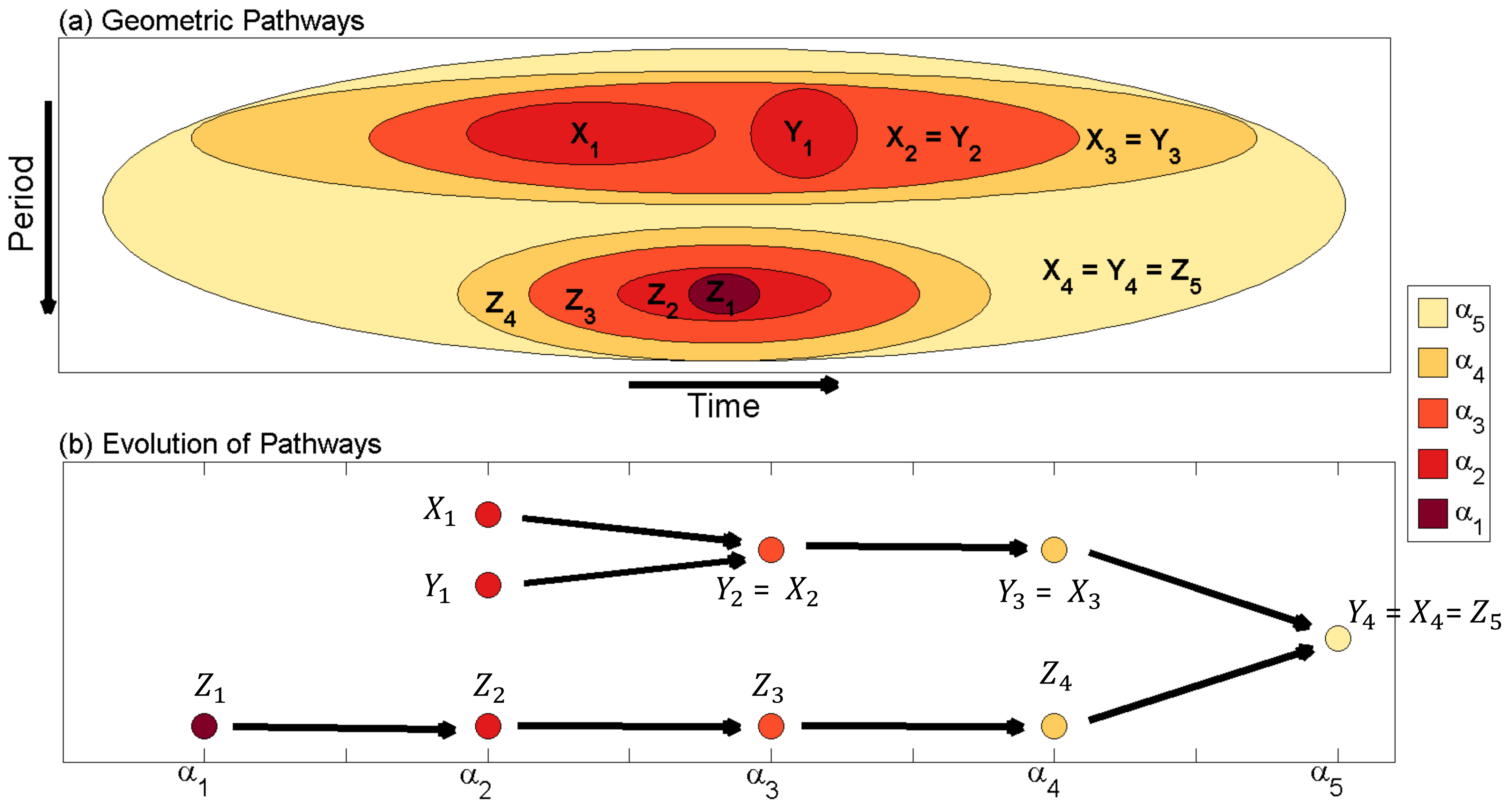
$$g_1 < g_2 < g_3 \ldots < g_r$$

$$g_i = \frac{A_i}{\left(C^i\right)^2}$$

A_i = area of patch

C_i = scale-coordinate of centroid

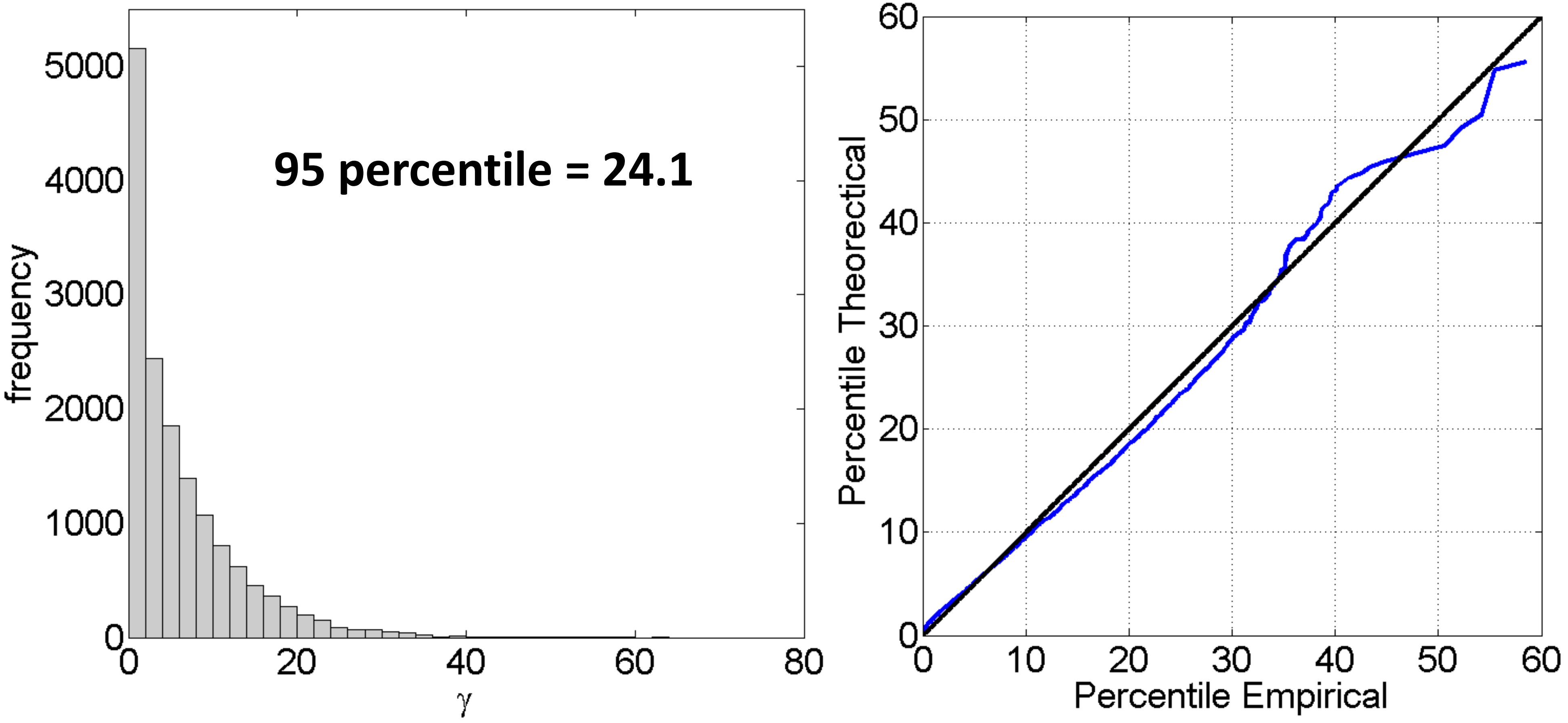
Geometric Pathway



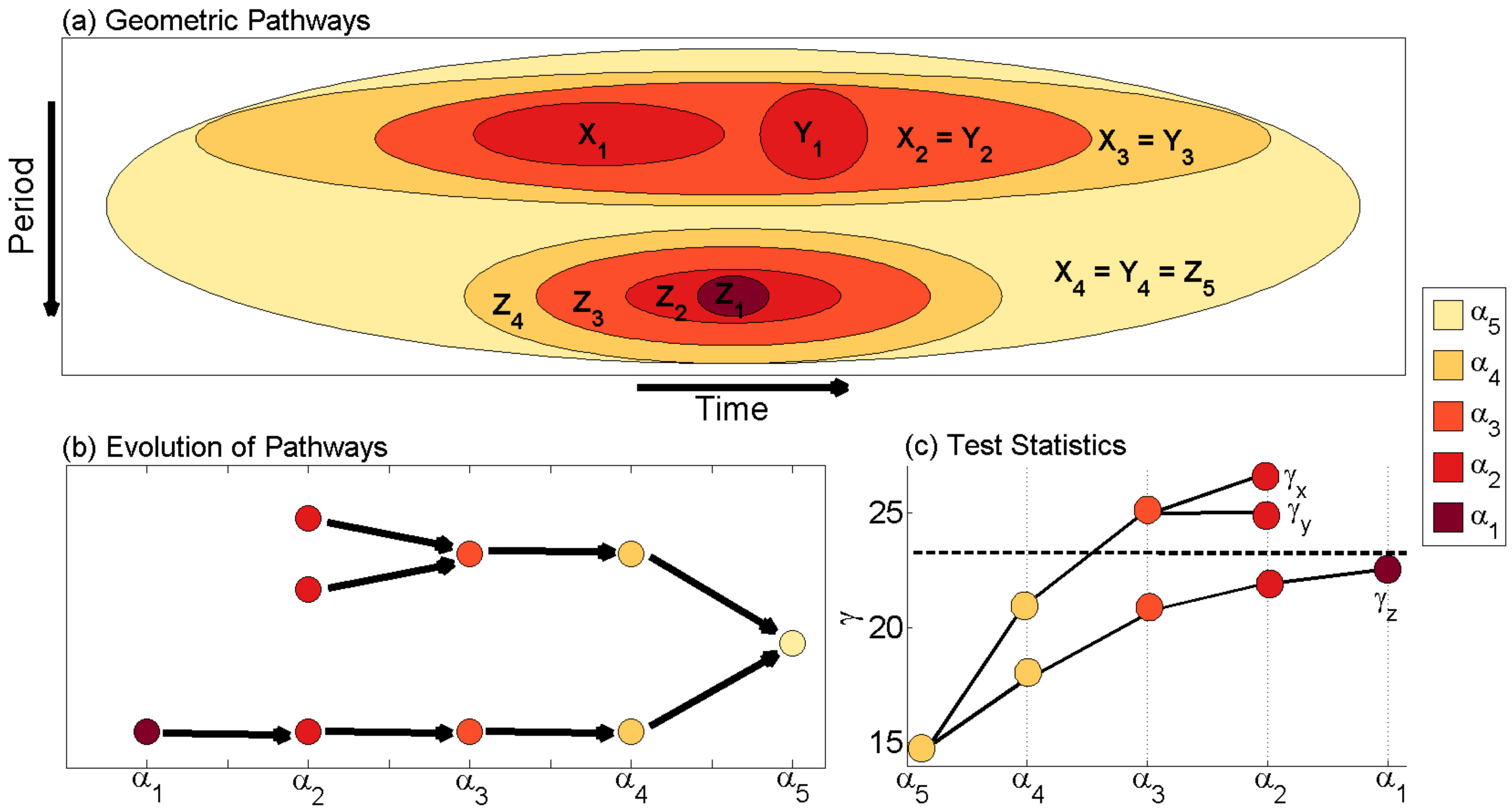
Test Statistic

- $\gamma = \sum_{j=1}^{r} g_j$ = $(sum\ of\ areas\ over\ all\ pathway\ elements)$
- γ is the sum of the g_j's over all elements of a pathway
- Calculate the critical level of the test using Monte Carlo Methods by generating a large ensemble of pathways under some noise hypothesis

The Null Distribution

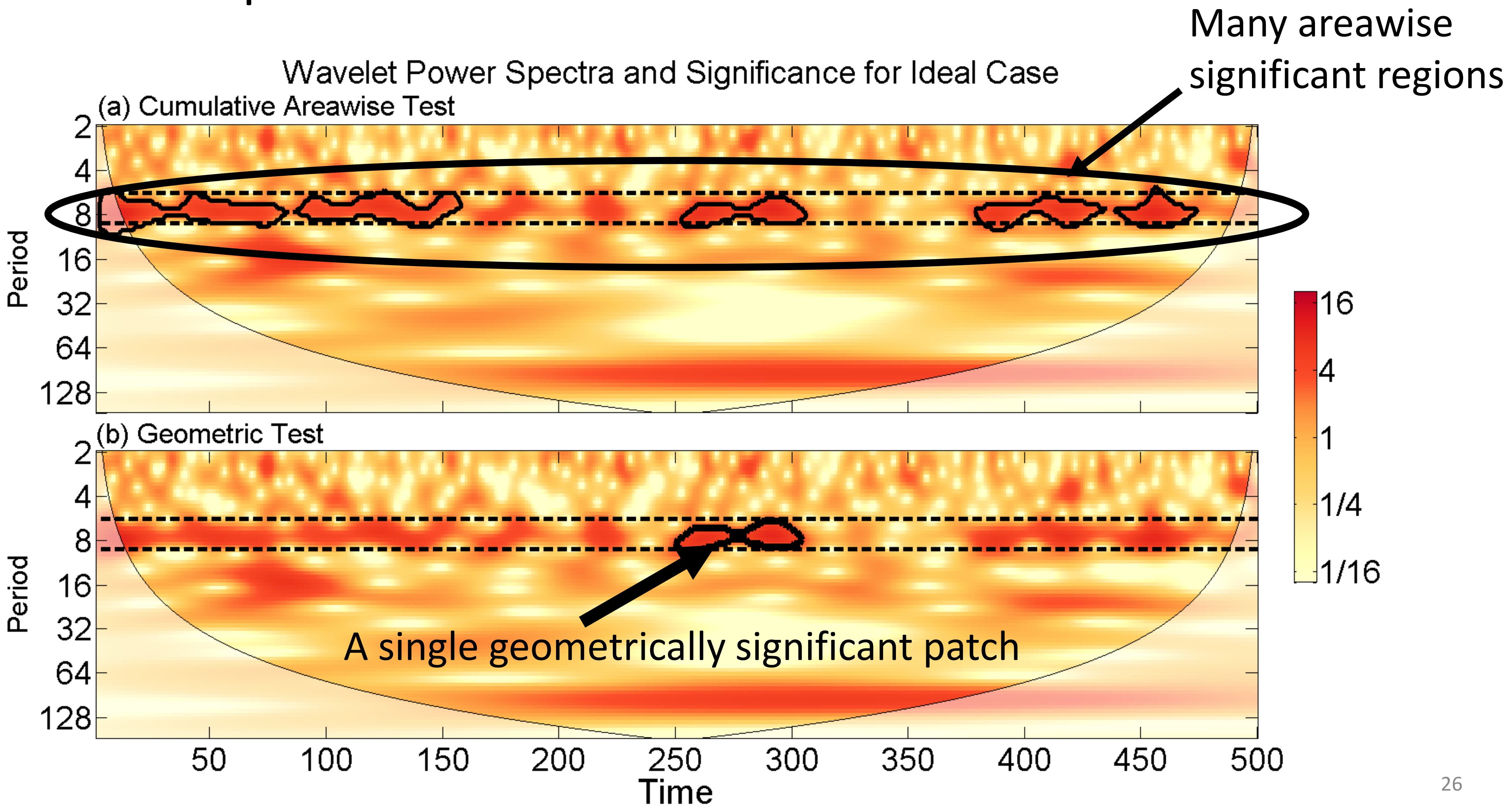


Geometric Pathways



Some experiments

$X(t) = A\sin(ft) + w(t)$



Experiment 1

Signal-to-noise

	0.5	1	5
α_{pw} = 0.05, α_{geo} = 0.05	0.25	0.29	0.54
α_c = 0.05	0.49	0.53	0.82
α_c = 0.01	0.43	0.49	0.82

$$X(t) = \sin(2\pi f t) + w(t)$$

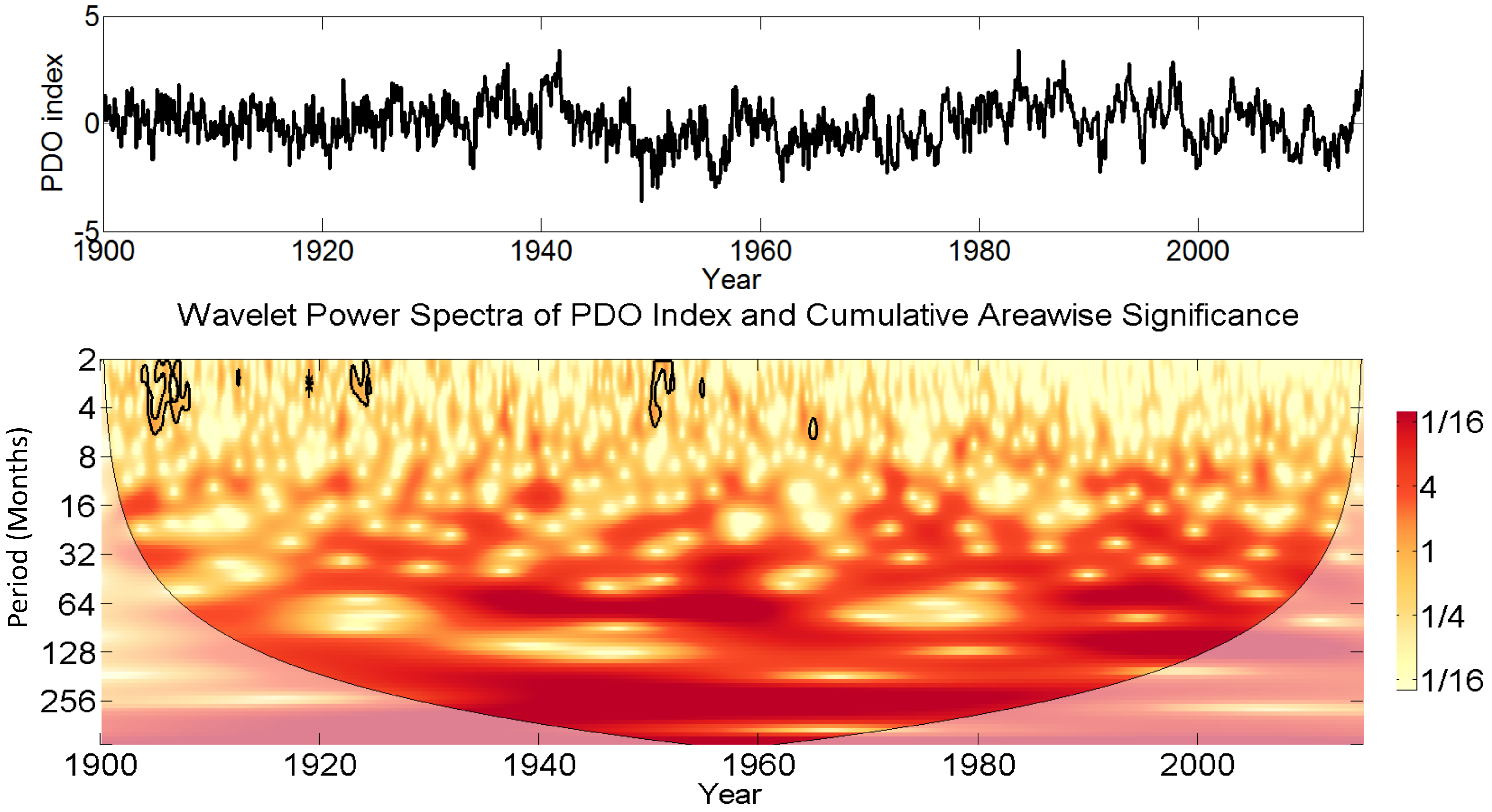
Experiment 2

Signal-to-noise

	0.5	1	5
α_{pw} = 0.05, α_{geo} = 0.05	0.15	0.18	0.43
α_c = 0.05	0.31	0.37	0.72
α_c = 0.01	0.22	0.27	0.68

$$X(t) = 0.6\sin(2\pi f t) + w(t)$$

Geophysical Example



Conclusions

- Cumulative test has greater statistical power than the existing geometric test
- PDO index during the period 1900-2014 was found to be indistinguishable from a red-noise background
- The PDO index may be the reddened response of stochastic white noise forcing and possibly the ENSO signal

Software Availability

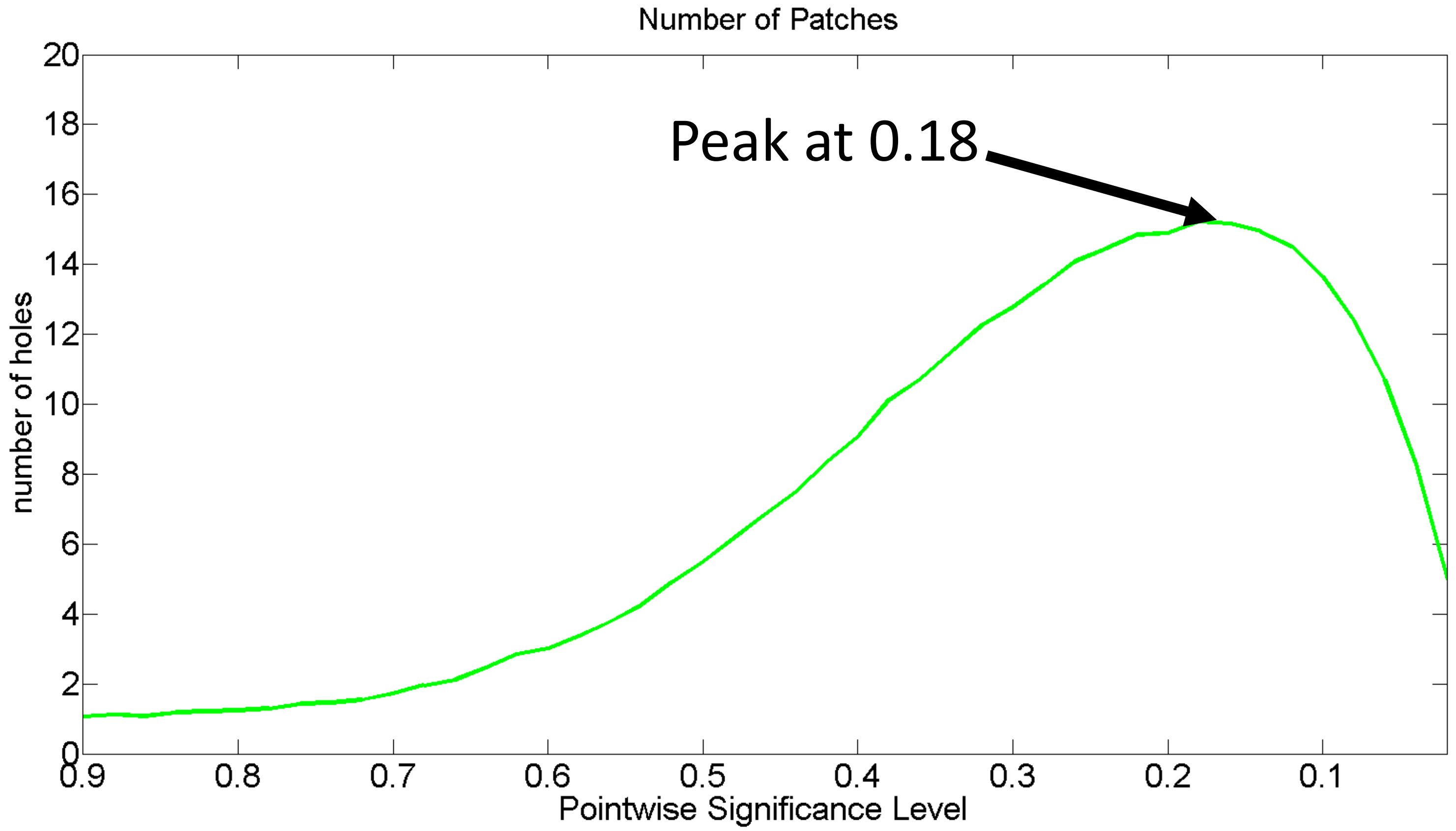
- Software for the geometric significance test can be accessed at justinschulte.com
- Cumulative areawise testing software will also be available upon publication

References

- Newman, M., Compo, G. P., Alexander, M. A.: ENSO-forced variability of the Pacific Decadal Oscillation. J. Climate, 16, 3853-3857, 2003.
- Maraun, D., Kurths, J., and Holschneider, M.: Nonstationary Gaussian processes in wavelet domain: synthesis, estimation, and significance testing, Phys. Rev. E, 75, doi: 10.1103/PhysRevE.75.016707, 2007.
- Schulte, J. A., Duffy, C., and Najjar, R. G.: Geometric and Topological Approaches to Significance Testing in Wavelet Analysis, Nonlin. Processes Geophys., 22, 139-156, 2015.
- Torrence, C. and Compo, G. P.: A practical guide to wavelet analysis, Bull. Amer. Meteor. Soc., 79, 61–78, 1998.

Extra Slides

Selecting pointwise significance levels



- Calculate statistical properties for $I = [0.01\ 0.18]$
- Recall that lifetimes of patches are mostly 0.01 so that the spacing between pointwise significance levels should also be 0.01.
- However, calculations showed that one can choose 0.02 without changing the properties of the test

Choosing the Output of the Testing Procedure

- Let $\gamma_j = \sum_{i=0}^{r-j} g_{r-i}$ so that, for example, for *r* = 4

$$y_1 = g_1 + g_2 + g_3 + g_4$$

$$y_2 = g_2 + g_3 + g_4$$

$$y_3 = g_3 + g_4$$

$$y_4 = g_4$$

- Use the element of the pathway associated with

$$\gamma_{\max} = \max_{j=1,2,\ldots,r} \gamma_j > \gamma_{crit}$$