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# Understanding Historical Streamflow Variability: The Wavelet Approach

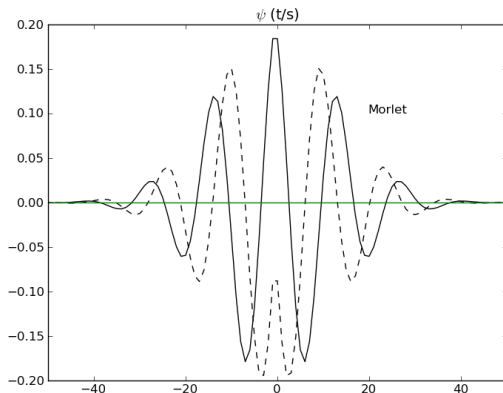
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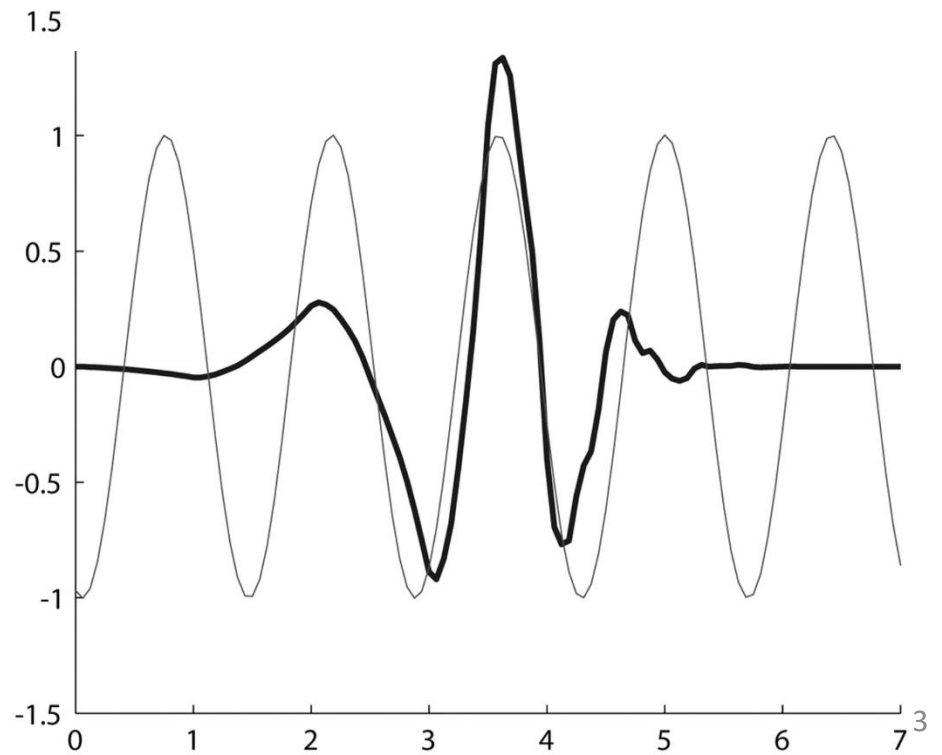


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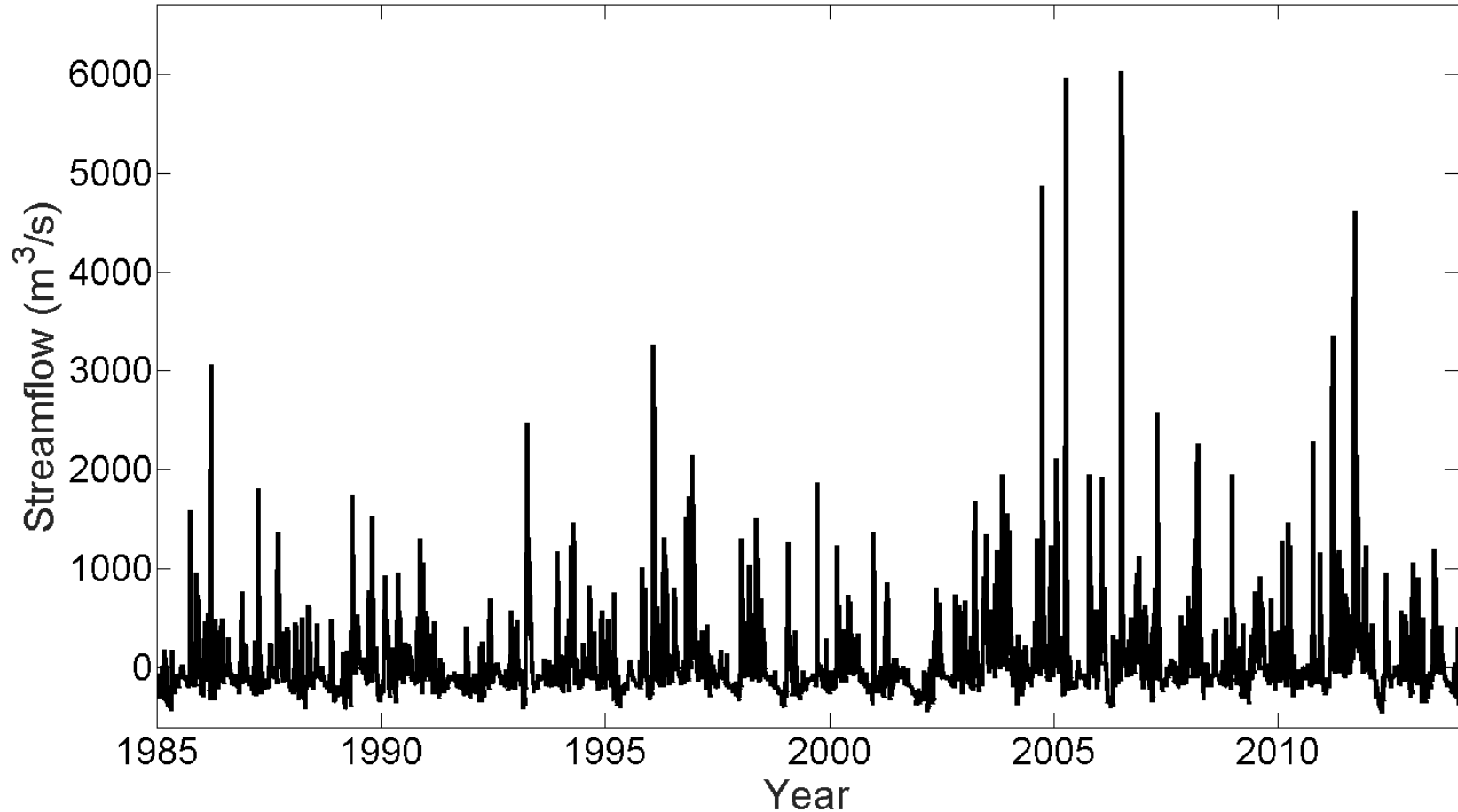
# Research Questions/Outline

- Can new wavelet methods help us understand historical streamflow variability?
- Can a new climate index be constructed that is better correlated with mid-Atlantic streamflow than existing climate indices?
- Was historical streamflow variability influenced by tropical convection?

# WAVELET ANALYSIS



# Delaware River Streamflow



# Wavelet Analysis

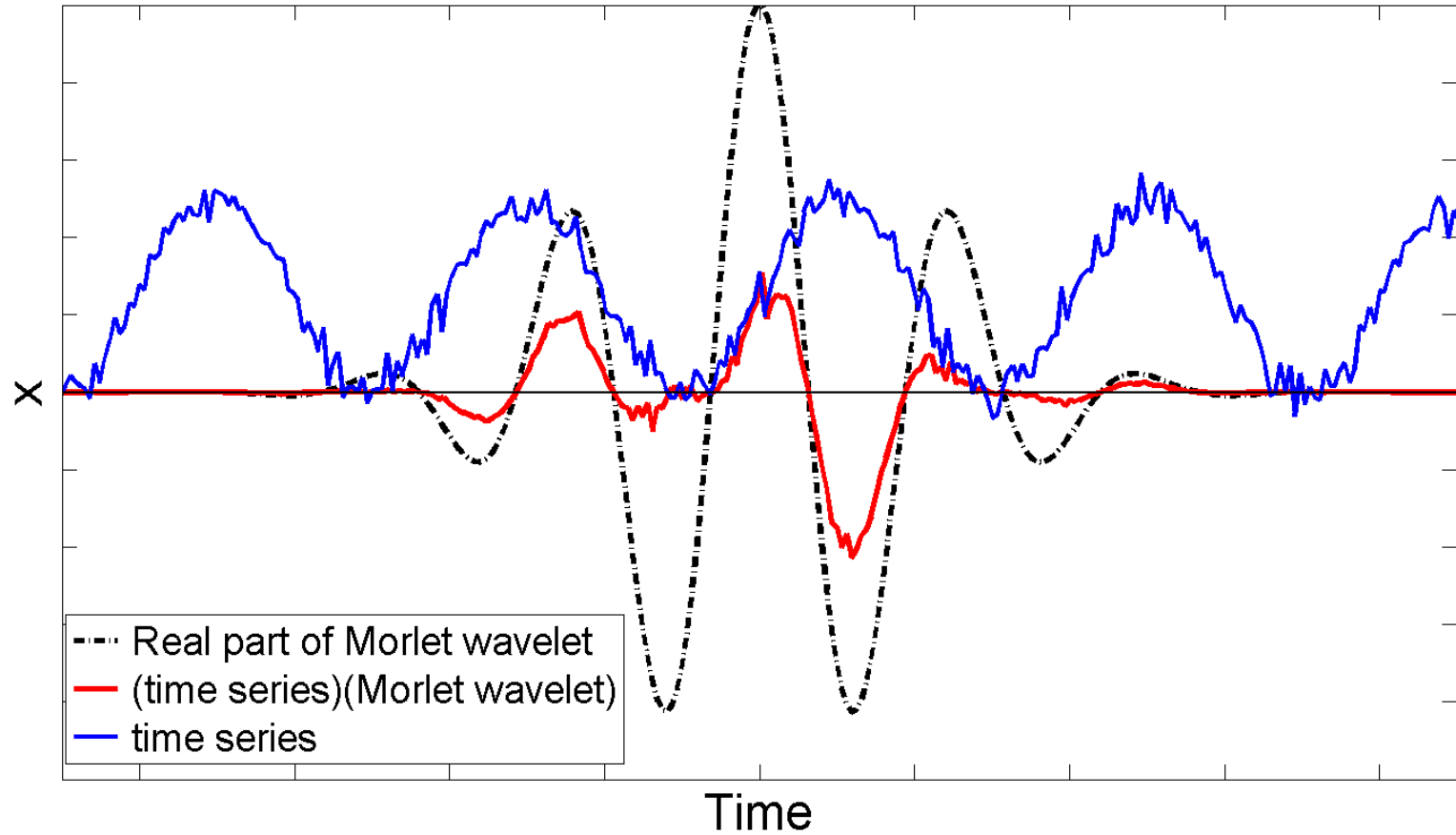
- Used to decompose variance (power) of a time series as a function of time and frequency
- Wavelet analysis can detect embedded oscillations in time series

# The Wavelet Transform

- $W_n^X(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^N x_{n'} \psi_0\left[(n' - n) \frac{\delta t}{s}\right]$
- $x_{n'}$  = time series
- $\delta t$  = time step determined from data
- $\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}$ ,
- Wavelet power =  $(W_n^X(s))^2$

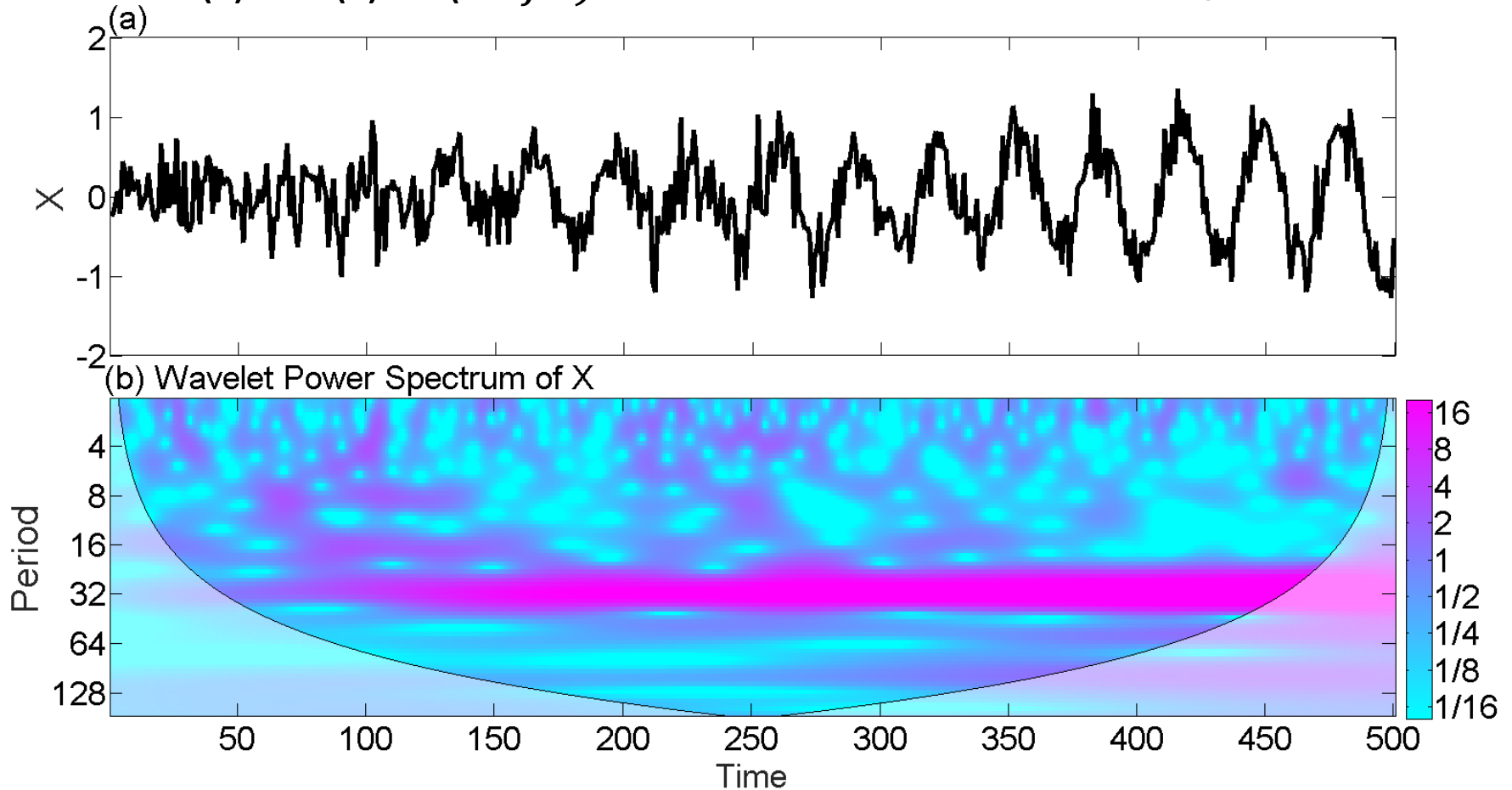
# The Wavelet Transform

Application of Wavelet Transform



# The Wavelet Power Spectrum

$$X(t) = A(t)\sin(2\pi ft) + \text{noise} \quad A(t) = t/500 \quad 2\pi f = 0.2$$

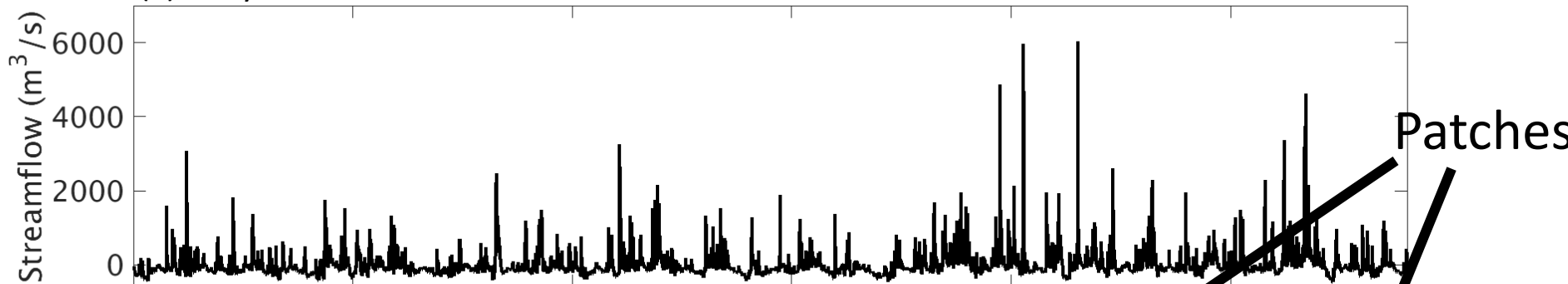




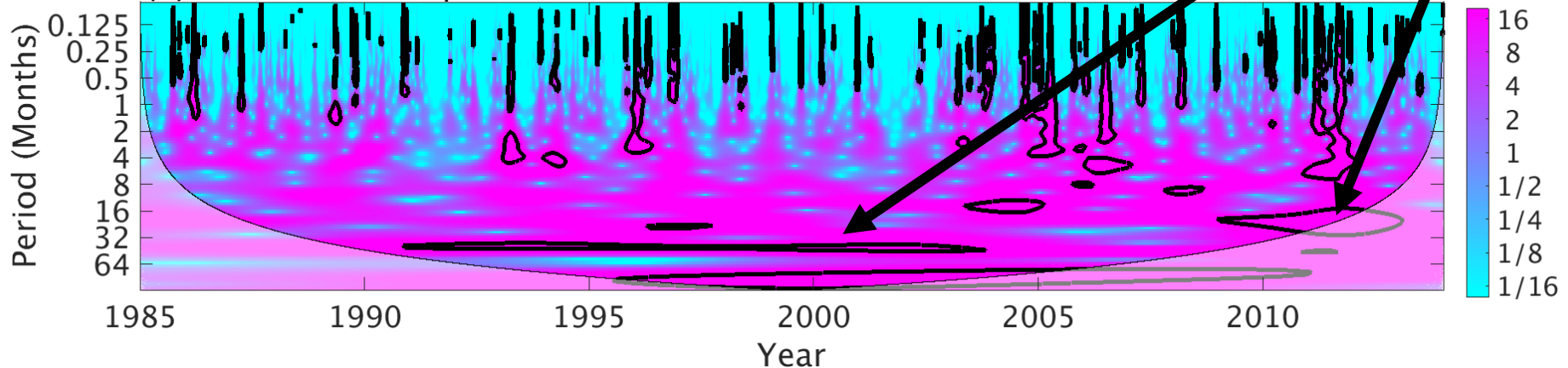
# Pointwise Significance Testing

- The wavelet power needs to be compared to that of a red-noise background.
- Red-noise can produce large wavelet power
- Tests each wavelet power coefficient independently

(a) Daily Delaware Streamflow



(b) Wavelet Power Spectrum of Delaware Streamflow



## Key Features

- Significant variability at periods less than 2 months due to weather events
- Significant low-frequency variability at a period of 4-years

# Cumulative Areawise Testing (Schulte, 2015)

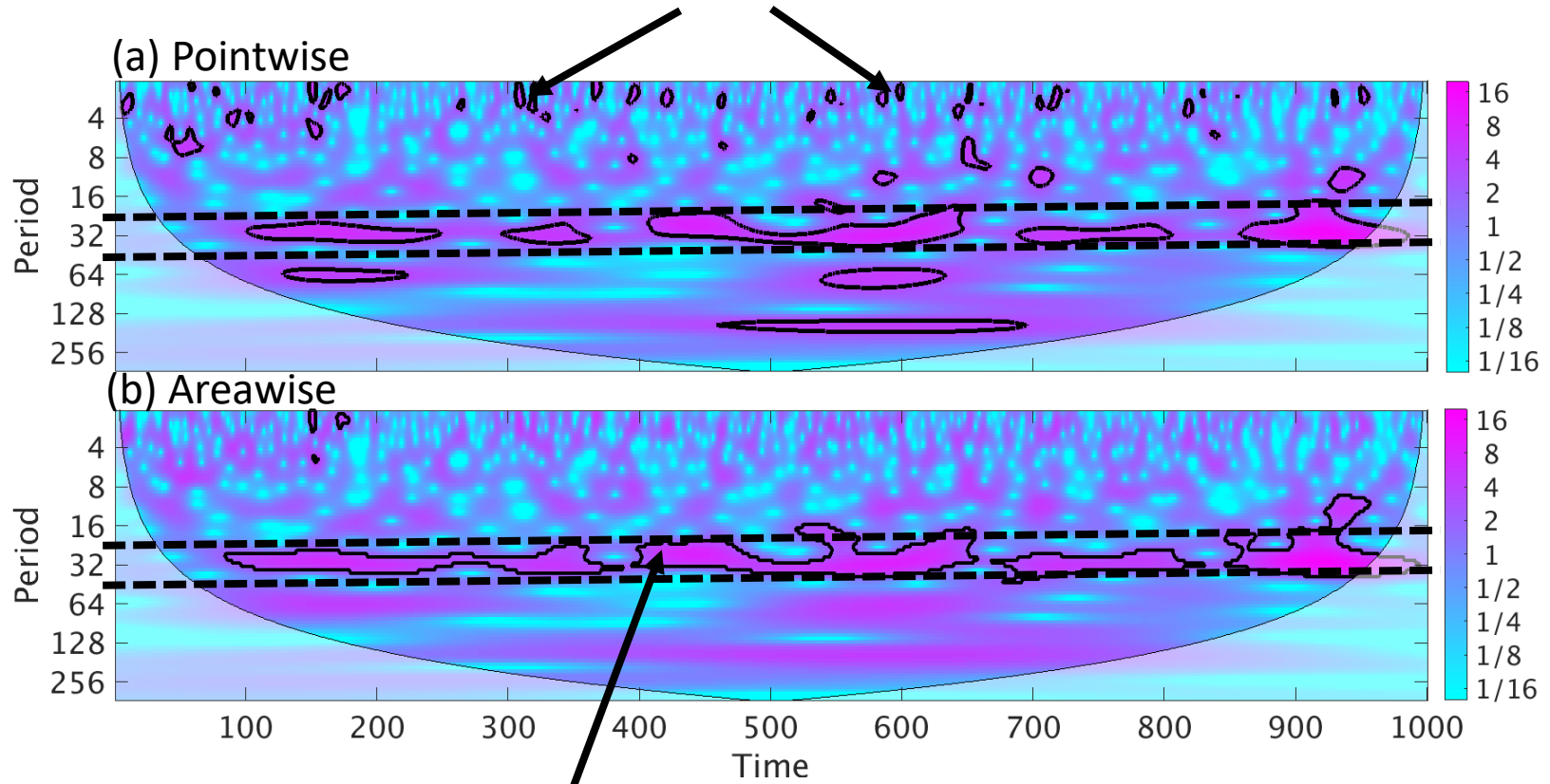
- Assesses the significance of patches based on **cumulative area** at different pointwise significance levels
- Uses ideas from persistent homology (Edelsbrunner, 2010)
- Reduces multiple-testing artifacts (Maraun et al., 2007)
- Researcher need not choose a pointwise significant level
- Test takes advantage of the autocorrelation among wavelet power coefficients

# Cumulative Areawise Testing

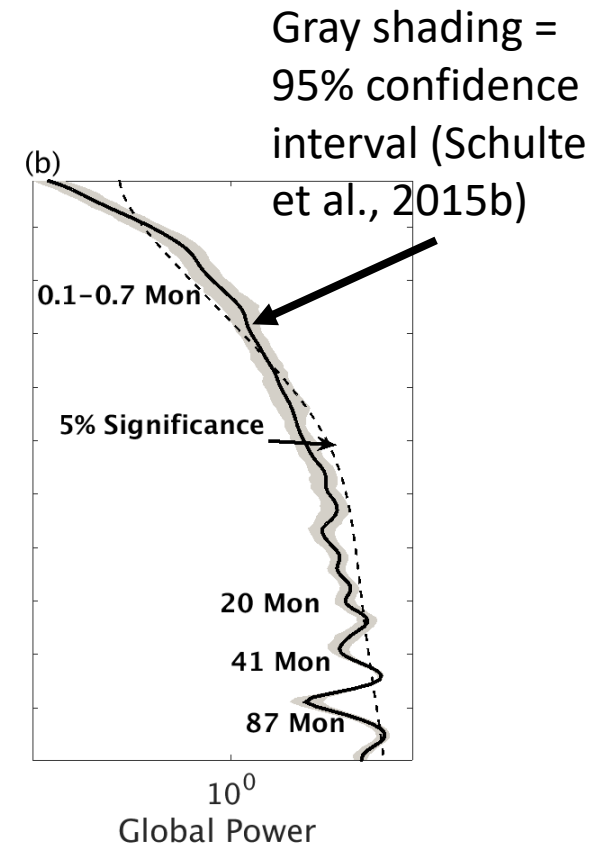
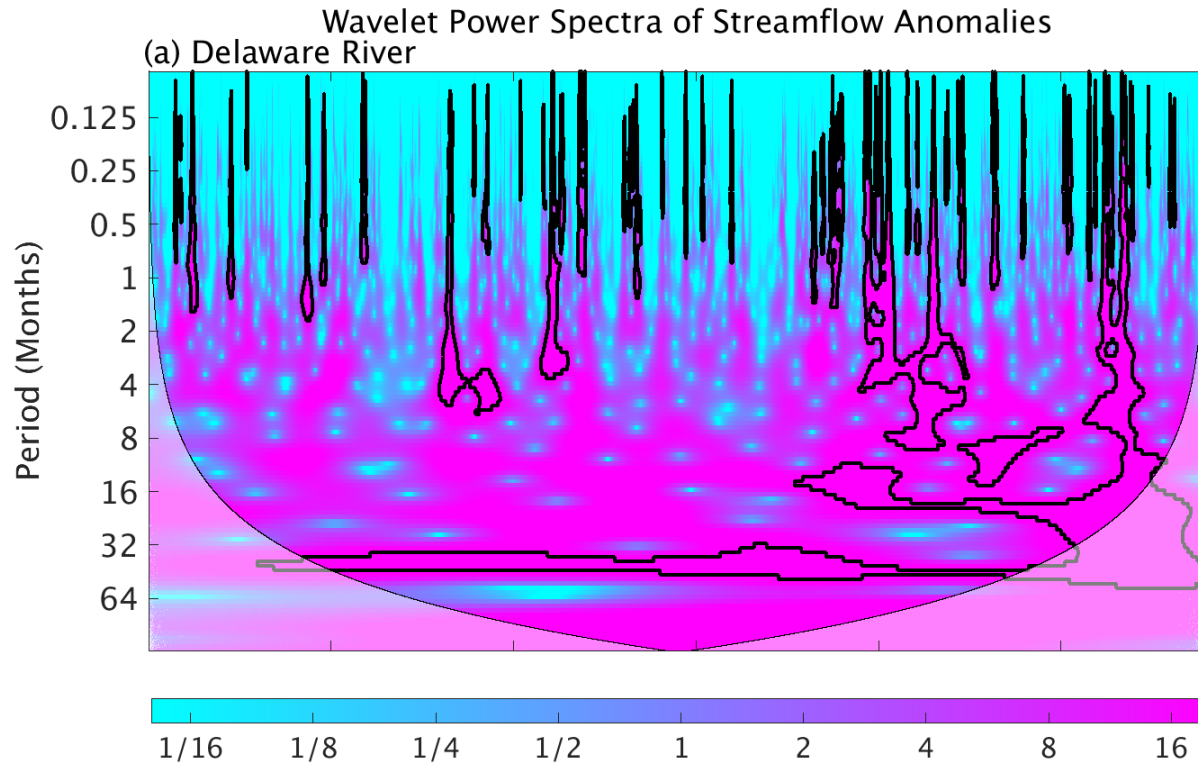
# Comparison of Tests

$$X(t) = \sin(2\pi f t) + \text{noise} \quad 2\pi f = 0.2$$

Anything outside dotted region are artifacts of multiple testing/spurious results.

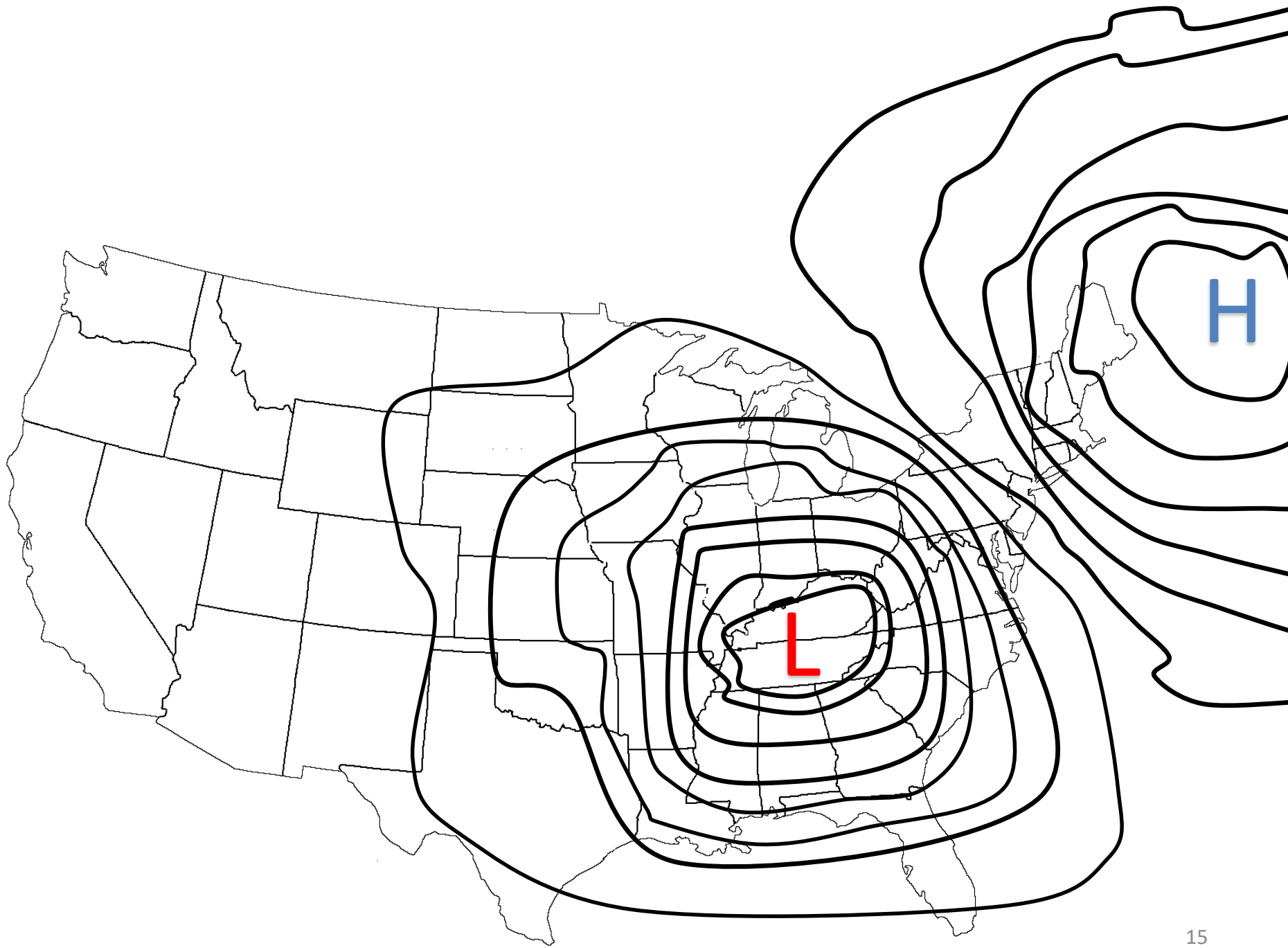


Anything within dotted region are true positives or what should be detected.



## Key Features

- Significant variability at periods less than 2 months
- Significant periodicity at a period of 2 years from 2003 through 2014
- Significant periodicity at a period of approximately 4 years

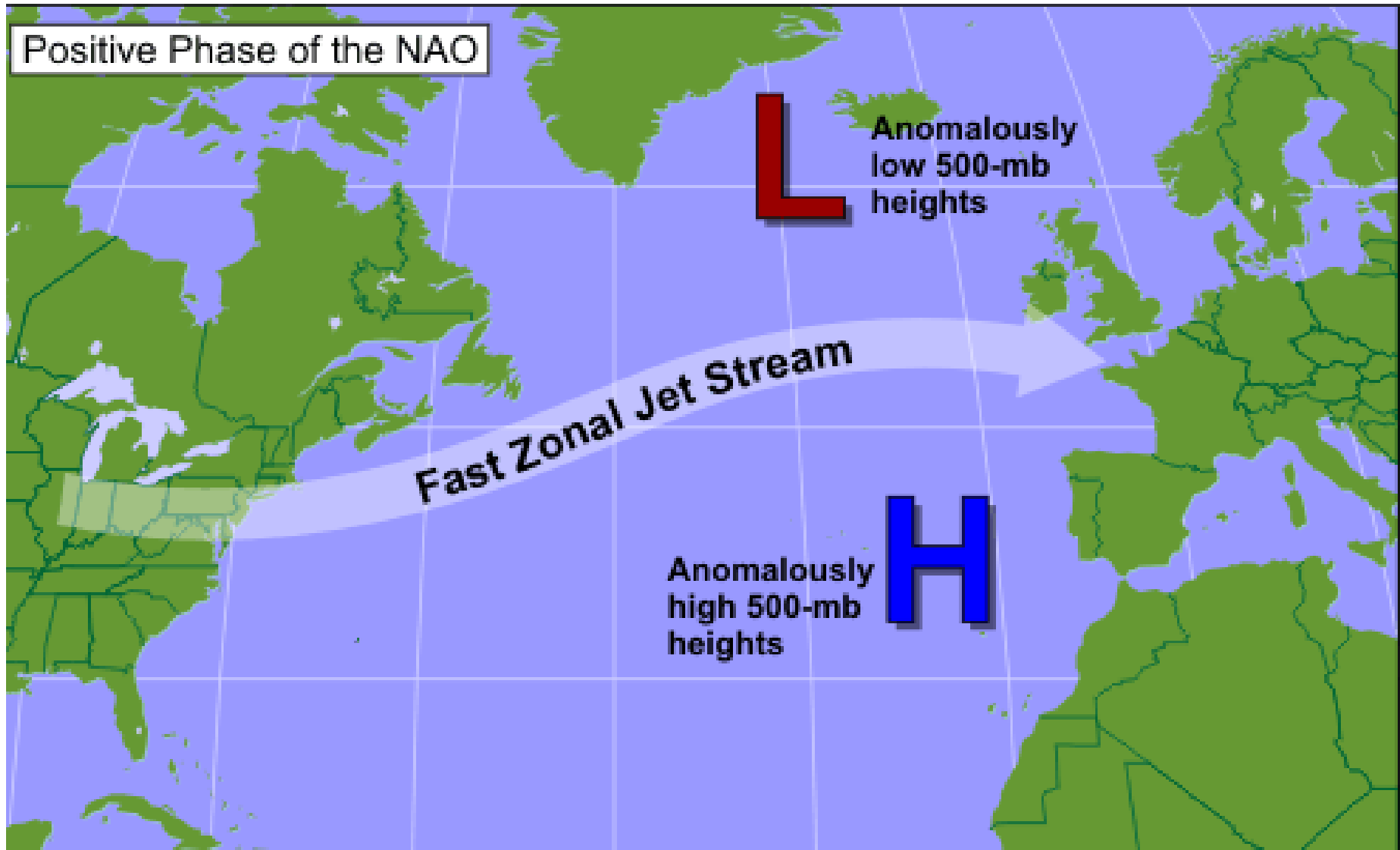


# Introduction

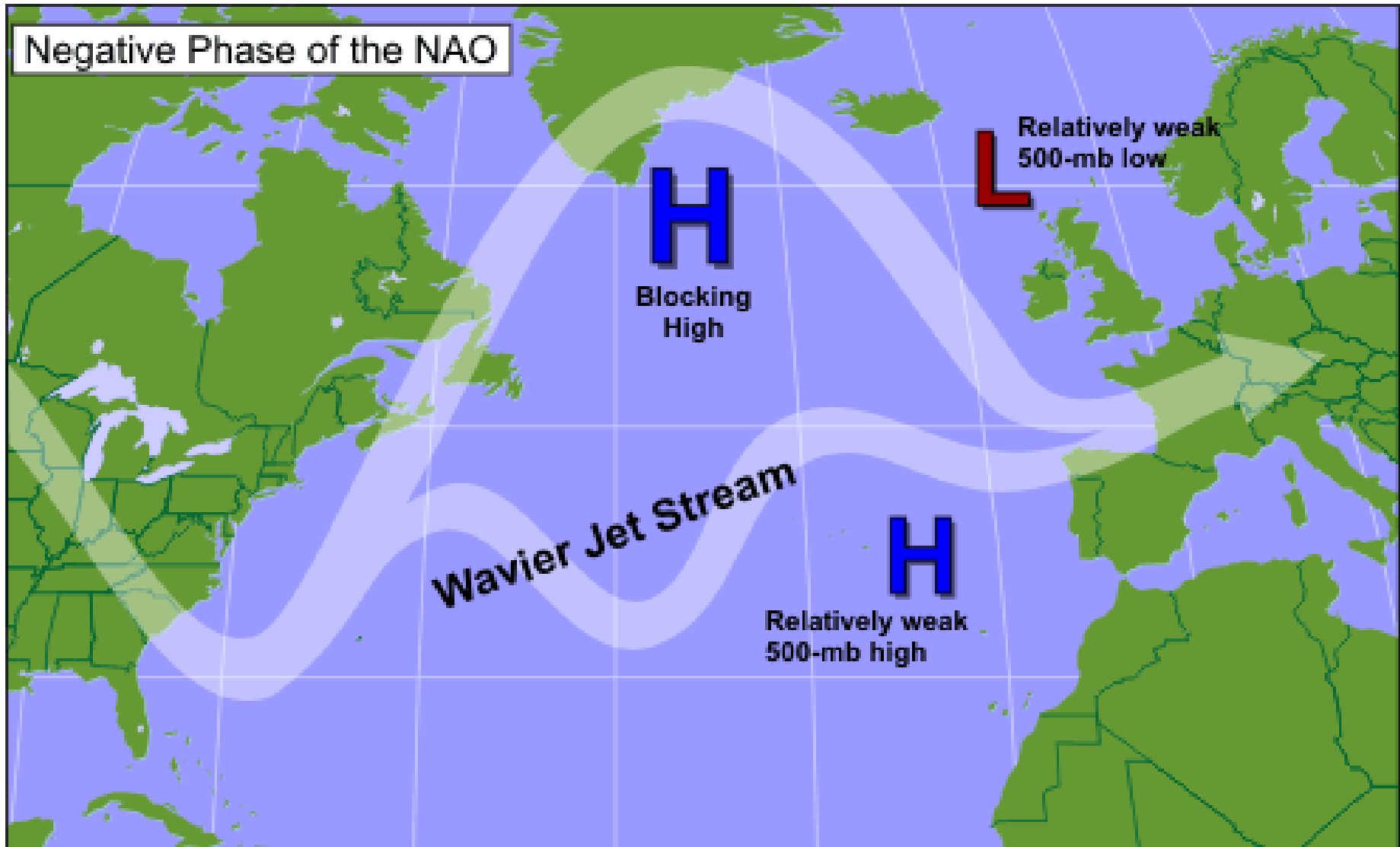
- Many studies have analyzed the impacts of the North Atlantic Oscillation (NAO) on precipitation and streamflow variability.
- It is useful to compare new climate indices to the well-studied NAO.



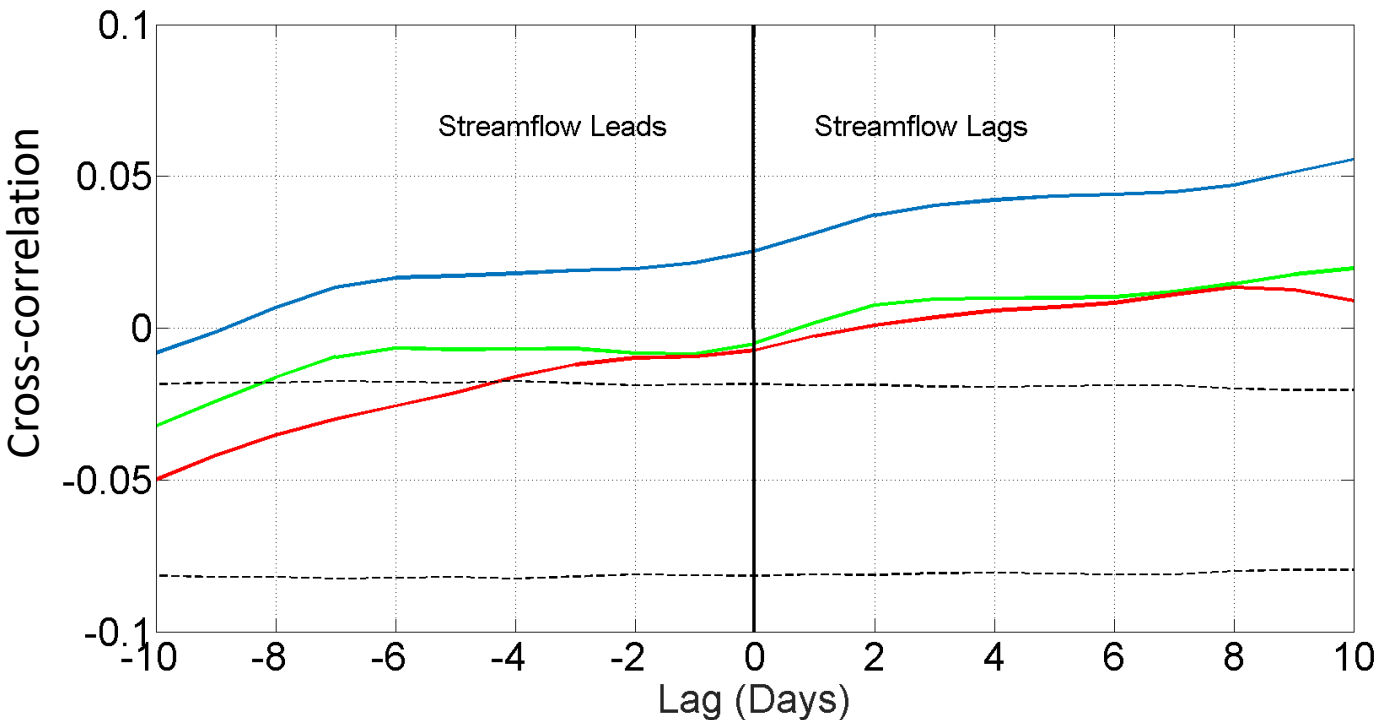
# North Atlantic Oscillation



# North Atlantic Oscillation



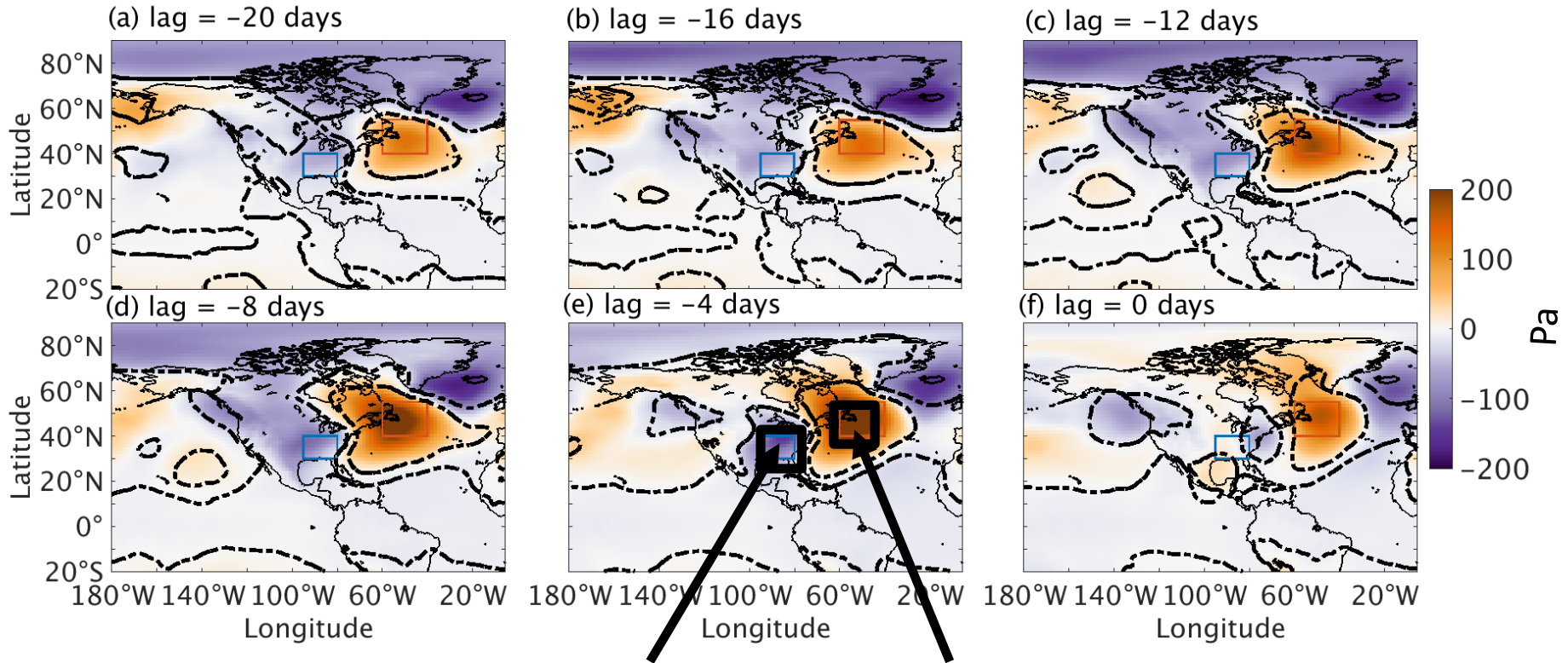
# Cross-correlation between Daily NAO Index and Daily Streamflow Anomalies



- NAO index is weakly but significantly correlated with streamflow
- Can we construct an index that is more correlated with mid-Atlantic streamflow

# The MSLP Dipole

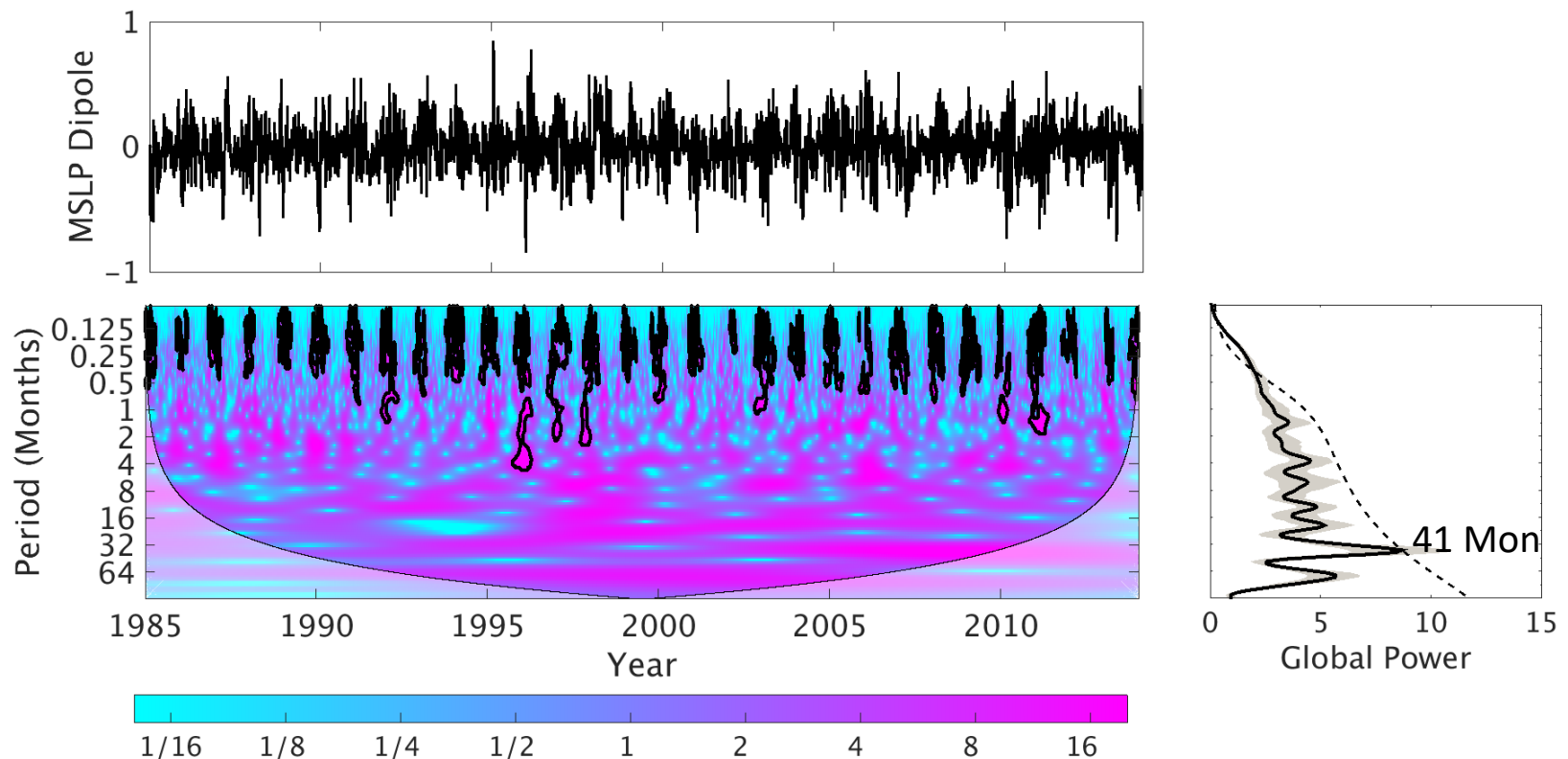
## Lag Composite Between Delaware River Streamflow and MSLP



Southeast MSLP Index    Atlantic MSLP Index

MSLP dipole = Atlantic MSLP Index - Southeast MSLP Index

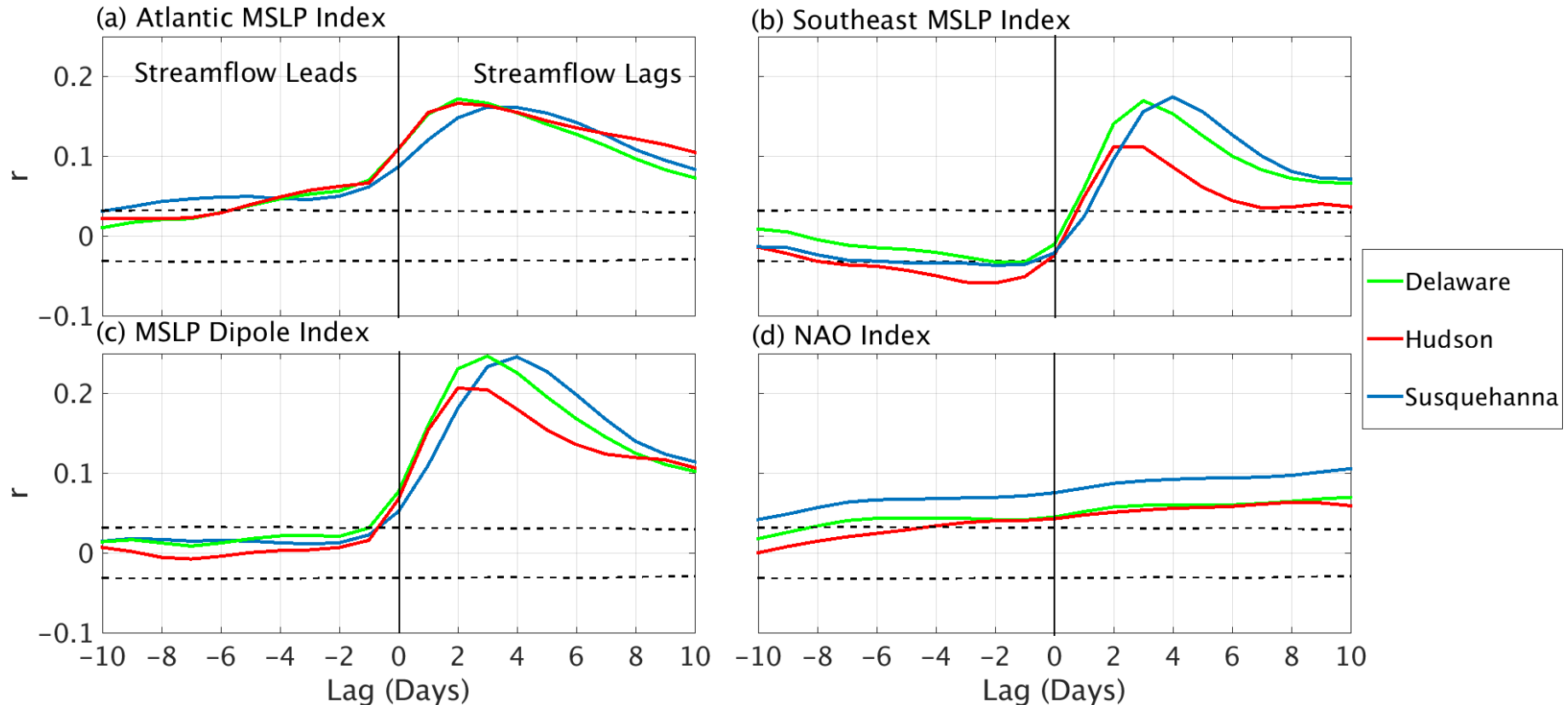
# The MSLP Dipole Index



## Key Features

- Significant variability at periods of 4-14 days
- Enhanced power at a period of 41 months

# Cross-correlation Analysis

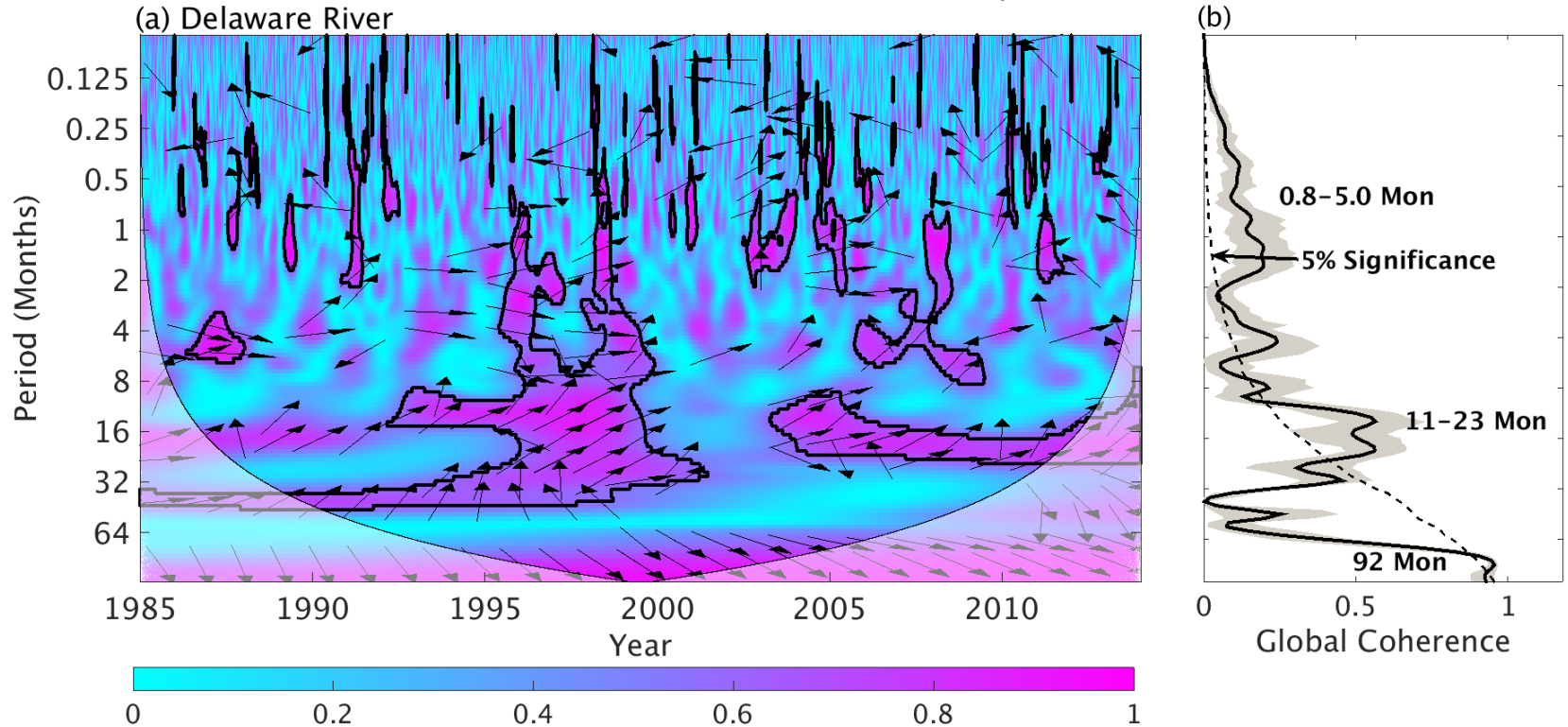


- All three new MSLP indices correlate better with streamflow than the existing NAO index

# Wavelet Coherence: Predictability Beyond the Weather Forecasting Timescale?

- Decomposes a correlation coefficient as a function of time and frequency (Grinsted et al., 2004)
- Wavelet coherence takes values between 0 and 1
- A value of 1 indicates the strongest possible relationship
- 0 means time series are **independent**
- Significance of wavelet coherence is assessed using Monte Carlo methods

## Wavelet Coherence between Streamflow and the MSLP Dipole Index

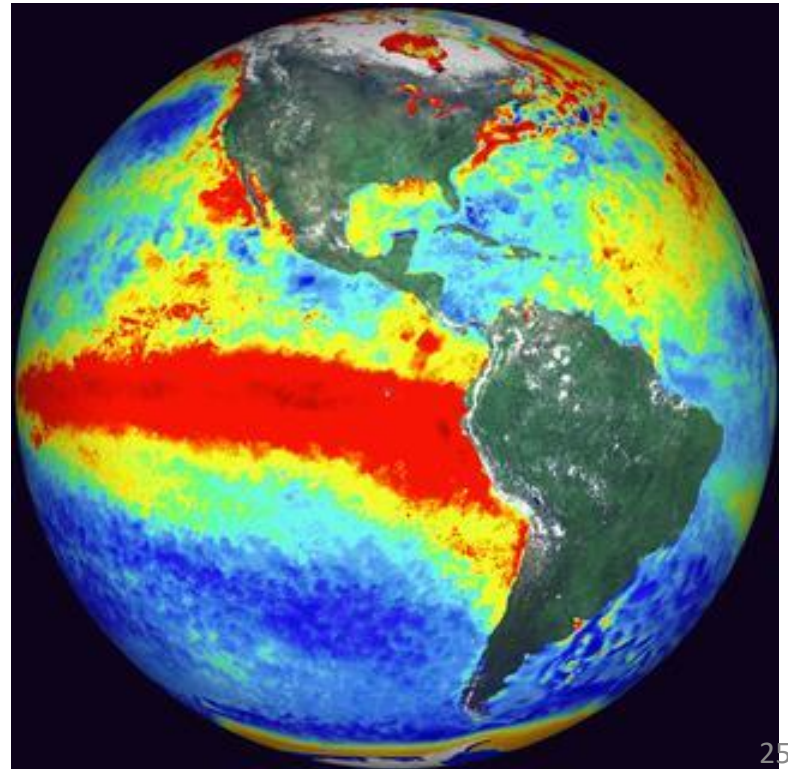


### Key Features

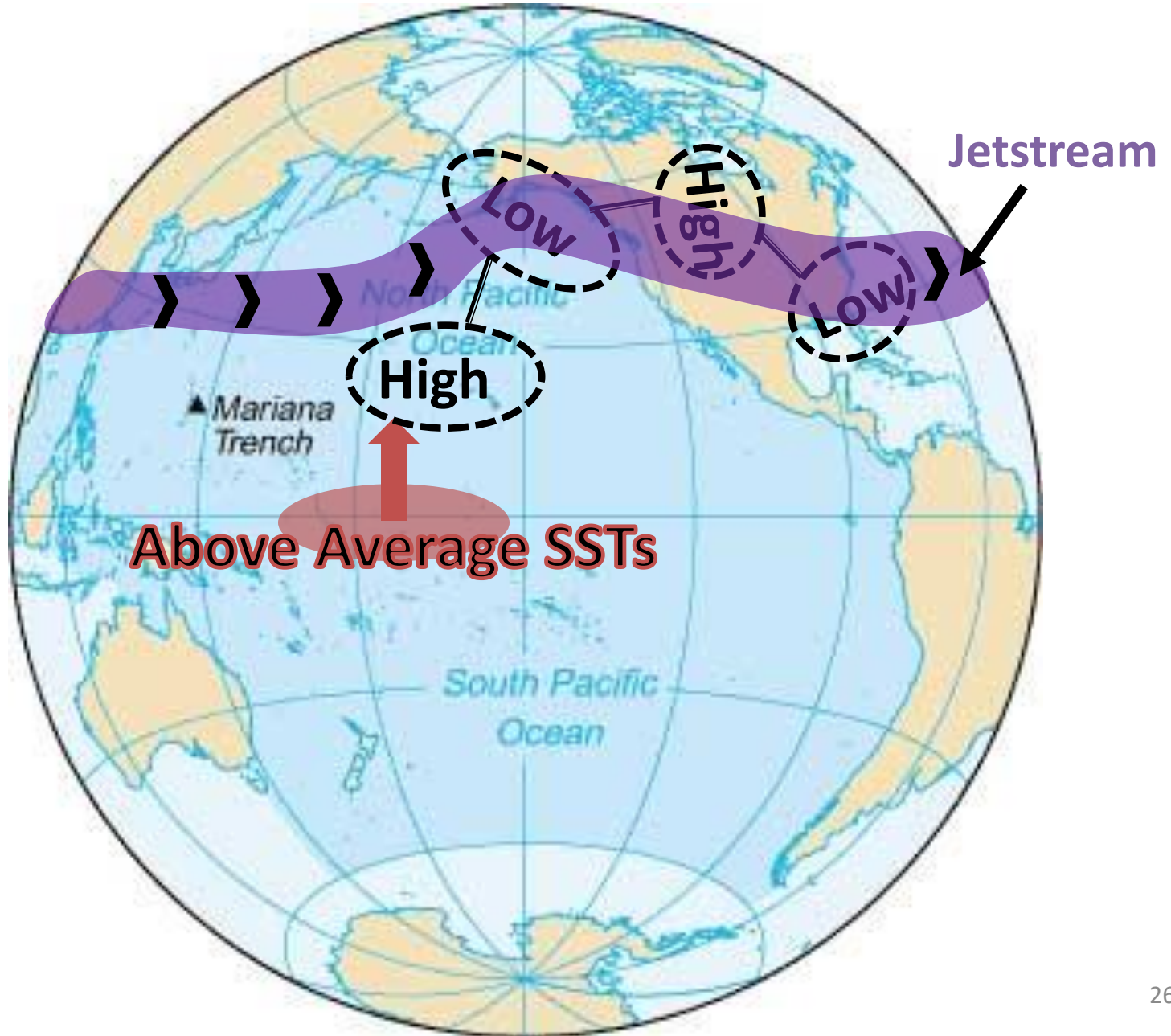
- Significant coherence at daily and seasonal timescales
- Significant coherence in the 1-4 year period band
- The coherence at 2 and 4 years coincides with significant streamflow variability (slide 14)



# TROPICAL INFLUENCES

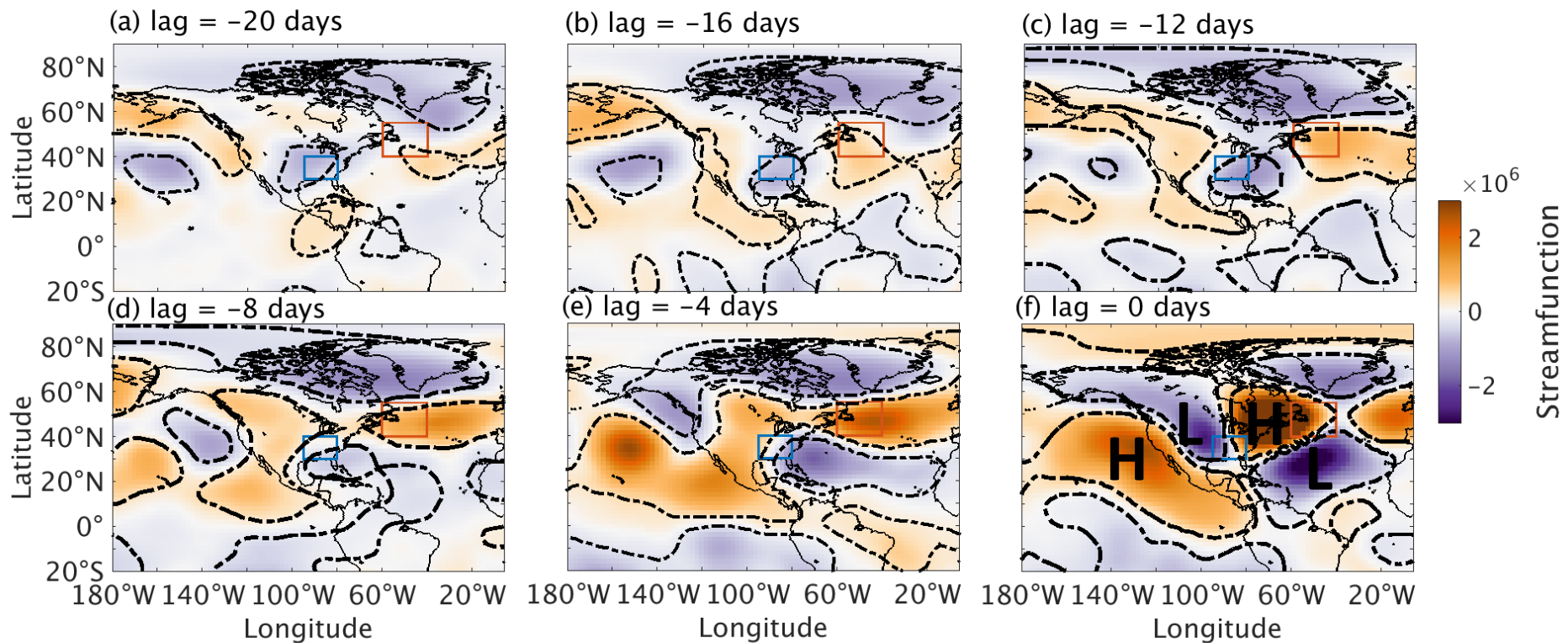


# Mid-latitude Response to Tropical Convection (Hoskins and Karoly, 1981)



# Tropical Convection and the MSLP Dipole Index

## Lag Composite Between MSLP Dipole Index and 300 hPa Streamfunction



# Conclusions

- New and powerful tools in wavelet analysis detected significant variability in the Delaware River time series at period of 2 and 4 years
- A new MSLP dipole index was constructed that is better correlated with mid-Atlantic streamflow than existing climate indices
- The MSLP dipole index may offer predictability beyond the weather forecasting timescale.
- The MSLP dipole index was related to tropical convection

# Software Availability

- Software for areawise testing and confidence interval calculations will be available upon publication
- Geometric significance testing (Schulte et al., 2015) software is available through [justinschulte.com](http://justinschulte.com)
- Please email me at [jas6367@psu.edu](mailto:jas6367@psu.edu) if you are interested in the software used in the presentation

# References

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